MODULAR SUPERCUSPIDAL LIFTS OF WEIGHT 2

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Abstract. One of the many problems within the Langlands functoriality conjecture is that of the automorphic base change, namely: given a number field extension L/K and an automorphic representation ρ of K, is it true that $\rho|_{\mathrm{G}_L}$ is still automirphic?. This problem was tackled first by Doi and Naganuma for real quatratic extensions of $\mathbb{Q}$ and generalised by Saito for totally real cyclic extensions. Further, Langlends, Arthur and Clozel tackled several instances of non-rational base fields. In 2012, Dieulefait proved in [3] that base-change works for any totally real extension of $\mathbb{Q}$. Later, in [2] he gave a simplified proof of this fact, jointly with a proof that the Sym^5 of a modular representation is still automorphic. Pacetti found a mistake in one of the many involved steps of the proof, which we have recently corrected in [1].

In the present work we will explain the proof of the first part of [2], namely; given a totally real number field $F/\mathbb{Q}$, $\mathfrak{N}$ an ideal of $\mathcal{O}_F$, a prime $p > \max\{k+1, 6\}$ totally split in F such that $p \nmid \mathrm{Norm}(\mathfrak{N})$ and non CM Hilbert cuspidal Hecke eigenform of level $\mathfrak{N}$ and parallel weight k, there exists a lift of $\overline{\rho}_{f,\mathfrak{P}}$ associated to a Hilbert modular cuspform $g \in S_{\mathbf{2}}(\mathfrak{N}p^2, \epsilon)$ which is supercuspidal at each prime of F over p. We also give a proof of the corresponding statement for classical Hecke cuspforms. Such statement was already proved by Khare with classical techniques. Hence our contribution, on one side, generalises Khare's result to Hilbert modular forms, making it a tool to tackle further instances of Langlands's functoriality and on the other side shows how the result for dimension one forms allows to fix the error found by Pacetti.

References

[1] Blanco-Chacón, I.: Dieulefait, D.: Modular supercuspidal lifts of weight 2. *J. Number Theory* (to appear). Arxiv version: https://arxiv.org/abs/2310.11522

[2] Dieulefait, L.: Automorphy of $Sym^5(GL(2))$ and base change (with Appendix A by R. Guralnick and Appendix B by L. Dieulefait and T. Gee). *J. Math. Pures et Appl.*, 104 (2015), p. 619–656.

[3] Dieulefait, L.V.: Langlands base change for GL(2). *Ann. of Math.*, 176 (2012), p. 1015–1038.