Solutions for the Muskat equation with quadratic growth at infinity

Omar Sánchez Antonio

Supervised by Ángel Castro & Daniel Faraco

Abstract

This thesis focuses on the study of special solutions to the Muskat equation for the case of two fluids with different densities. This equation describes the evolution of the interface separating the fluids in a porous medium. In the first part of the thesis, we consider special solutions of the form

$$h(x,t) = x^2 + ct + g(x,t),$$

where g is in a suitable Sobolev space. In particular, this type of solutions grow quadratically at infinity. As far as we know this is the highest growth at infinity of solutions to the Muskat for which local existence has been proven.

The second part of the thesis focuses on the search of *turning* singularities for solutions with quadratic growth. In a *turning* singularity the solution starts in the stable regime, thus the function can be parameterize by the graph of a function. Then, at some finite time, the solution turns, and it can not be parameterized as the graph of function.

We prove that there are solutions to the Muskat equation that grow quadratically at the infinity and develop a *turning* singularity. In particular this implies that global solutions can not be achieved.

In the final part of the thesis we consider a modification of the Muskat equation that takes into account surface tension. This new force is introduced through a jump discontinuity of the pressure across the interface proportional to the curvature. In this case, we do not deal with solutions that grow quadratically at the infinity, but with asymptotically flat interfaces. We provide a description of the stationary solutions for the Muskat problem with surface tension, looking for 2π-periodic solutions.