## Eigenfunctions and inverse localization

A well-known link between solutions to the Helmholtz equation $\Delta v + v = 0$ and eigenfunctions of a Schrödinger operator on a Riemannian manifold, $\Delta_g u + \lambda^2 u - Vu = 0$, is that the local behaviour of a sequence of high-energy eigenfunctions (say, $\lambda^2 \to \infty$) defines a bounded Helmholtz solution, after suitable rescalings. Conversely, every solution to Helmholtz can be locally realized by an approximate eigenfunction of any large enough energy, on scales determined by this energy.

A powerful refinement of the latter fact is what we call the inverse localization principle: if, roughly speaking, the degeneracy of the high-energy eigenvalues is large enough, one can replace the quasimodes by *bona fide* eigenfunctions.

In this talk we introduce the inverse localization principle and prove a precise version of this theory for the harmonic oscillator potential in $\mathcal{H}^d(\kappa)$, the hyperbolic space of constant curvature $-\kappa^2$. Finally, we mention some key ideas behind the localization principle in order to develop a systematic theory, e.g. the underlying symmetry hypothesis.