On twisted ℓ^2-Betti numbers

ℓ^2-Betti numbers were originally defined as the analogous of normal Betti numbers for G-CW-complexes for a group G. They measure the von Neumann dimension of the cellular homology groups of a finite type free G-CW-complex with local coefficients in $\ell^2(G)$. A more algebraic way to study ℓ^2-Betti numbers is to study Sylvester matrix rank functions on $\mathbb{C}[G]$. In this talk we will explain how to twist a matrix $A \in \mathrm{Mat}_n(\mathbb{C}[G])$ with a finite dimensional representation $\varphi : G \to \mathrm{GL}_m(\mathbb{C})$. This gives us a new matrix $\tilde{A} \in \mathrm{Mat}_{m\cdot n}(\mathbb{C}[G])$. We will show that

$$\mathrm{rk}_G(\tilde{A}) = m \cdot \mathrm{rk}_G(A)$$