Resumen:
SIO (Singular Integral Operator) theory and, Calderón-Zygmund theory specifically, developed starting from the ‘60s, provides a vast array of tools for dealing with operators that resemble the Hilbert transform
\[ Hf(x) := \frac{1}{\pi} \int_{\mathbb{R}} f(x-y) \frac{dy}{y}, \]
an ubiquitous operator in Complex Analysis, semi-linear PDEs, and many other branches of mathematics. Results valid for -valued functions were extended to Banach spaces-valued functions thanks to Bourgain’s groundbreaking work on the deep relation between Banach space geometry and boundedness properties of vector-valued SIOs. Scalar-valued bounds for multilinear SIOs, like the bilinear Hilbert transform
\[ \text{BHT}[f_1, f_2](x) = \int_{\mathbb{R}} f_1(x-t)f_2(x+t) \frac{dt}{t}, \]
are classic in time-frequency-scale analysis. Banach-space valued results have appeared only in the last couple of years. The well understood connections with Banach space geometry from linear theory are just starting to be investigated. Open questions and generalizations to non-commutative analysis abound and would come hand-in-hand with progress in understanding SIOs with worse singularities than of Calderón-Zygmund type that can often be realized as SIO-valued CZ operators.

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