WORKING GROUP IN NONCOMMUTATIVE HARMONIC ANALYSIS

Martes, 3 de marzo de 2015

11:00-13:00 h., Aula Azul (ICMat, Campus de Cantoblanco)

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Contractivity and complete contractivity for balls in finite dimensional Banach Spaces

Resumen:

It is known that contractive homomorphisms of the disc and the bi-disc algebra to the space $\mathcal{B}(\mathcal{H})$ of bounded linear operators on a Hilbert space $\mathcal{H}$ are completely contractive, thanks to the dilation theorems of B. Sz.-Nagy and Ando respectively. Examples of contractive homomorphisms of the (Euclidean) ball algebra which are not completely contractive was given by G. Misra. From the work of V. Paulsen and E. Ricard, it follows that if $m \geq 3$ and $\mathbb{B}$ is any ball in $\mathbb{C}^m$ with respect to some norm, say $\|\cdot\|_{\mathbb{B}}$, then there exists a contractive linear map $L : (\mathbb{C}^m, \|\cdot\|_{\mathbb{B}}) \to \mathcal{B}(\mathcal{H})$ which is not complete contractive. The characterization of those balls in $\mathbb{C}^2$ for which contractive linear maps are always completely contractive remained open. We answer this question for balls of the form $\Omega_{\mathbf{A}}$ in $\mathbb{C}^2$ which are of the form

$$\Omega_{\mathbf{A}} = \{z = (z_1, z_2) : \|z\|_{\mathbf{A}} = \|z_1 A_1 + z_2 A_2\|_{\text{op}} \leq 1\}$$

for some choice of an 2-tuple of 2×2 linearly independent matrices $\mathbf{A} = (A_1, A_2)$. Let $\mathcal{O}(\Omega_{\mathbf{A}})$ denote the algebra of functions holomorphic on a neighborhood of the closed unit ball $\bar{\Omega}_{\mathbf{A}}$.

Theorem. *Suppose A_1 and A_2 are not simultaneously diagonalizable, that is, there does not exist a 2×2 unitary matrix U such that both matrices UA_1U^* and UA_2U^* are diagonal. Then the following holds:*

(i) *There exists a contractive homomorphism from $\mathcal{O}(\Omega_{\mathbf{A}})$ to $\mathcal{B}(\mathcal{H})$, which is not completely contractive, where $\mathcal{H}$ is a finite dimensional Hilbert space.*

(ii) *There exists a contractive linear map L from $(\mathbb{C}^2, \|\cdot\|^*_{\mathbf{A}})$ to $\mathcal{M}_{p,q}(\mathbb{C})$, which is not completely contractive. Here $\|\cdot\|^*_{\mathbf{A}}$ is the norm, dual to $\|\cdot\|_{\mathbf{A}}$.*

In fact, we show that one can find homomorphism in (i), which has the form

$$\rho_V(f) := \begin{pmatrix} f(w)I_p & \sum_{i=1}^2 \partial_i f(w) V_i \\ 0 & f(w)I_q \end{pmatrix}, \qquad f \in \mathcal{O}(\Omega_{\mathbf{A}}),$$

where V_1, V_2 are some $p \times q$ matrices.