UC3M–ICMAT Seminar – 2014/2015

Applied Probability and Statistics

Computing spectra, the Maslov index and nonlinear PDEs

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We address the problem of computing the Maslov index for large linear symplectic systems on the real line. The Maslov index measures the signed intersections (with a given reference plane) of a path of Lagrangian planes. The natural chart parameterization for the Grassmannian of Lagrangian planes is the space of real symmetric matrices. Linear system evolution induces a Riccati evolution in the chart. For large order systems this is a practical approach as the computational complexity is quadratic in the order. The Riccati solutions, however, also exhibit singularities (which are traversed by changing charts). Our new results involve characterizing these Riccati singularities and two trace formulae for the Maslov index as follows. First, we show that the number of singular eigenvalues of the symmetric chart representation equals the dimension of intersection with the reference plane. Second, the Cayley map is a diffeomorphism from the space of real symmetric matrices to the manifold of unitary symmetric matrices. We show the logarithm of the Cayley map equals the arctan map (modulo $2\mathrm{i}$) and its trace measures the angle of the Langrangian plane to the reference plane. Third, the Riccati flow under the Cayley map induces a flow in the manifold of unitary symmetric matrices. Using the natural unitary action on this manifold, we pullback the flow to the unitary Lie algebra and monitor its trace. This avoids singularities, and is a natural robust procedure. We demonstrate the effectiveness of these approaches by applying them to a large eigenvalue problem. We also discuss the extension of the Maslov index to the infinite dimensional case.

This is joint work with Margaret Beck (BU).

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