

Speaker: Haluk Sengun

Title: Torsion homology of 3-manifolds groups

The goal of this mini-course is to present the conjectures of Bergeron and Venkatesh on the asymptotic growth of torsion in the homology of congruence arithmetic groups in the special case of arithmetic Kleinian groups. We will start the mini-course by sketching the construction of arithmetic Kleinian groups from orders in quaternion algebras. We shall then consider the cohomology of arithmetic Kleinian groups together with the action of Hecke operators. In order to motivate the importance of torsion in the homology of arithmetic groups, we will discuss the partly-conjectural connection between cohomology of arithmetic Kleinian groups and Galois representations. Afterwards we will state and discuss the conjecture of Bergeron and Venkatesh. Time permitting we will also present some numerical data related to the conjecture.

Suggested Reading (all titles can be accessed from the homepages of the authors):

For the construction of arithmetic Kleinian groups:

- (1) Haluk Sengun: “Torsion in the Homology of Arithmetic Kleinian Groups” (compact outline of the construction)
- (2) Kathleen Petersen: “Arithmetic Groups and Lehmer’s Conjecture” (slightly more detailed outline of the construction)
- (3) C. Maclachlan - A.Reid: (book) “The Arithmetic of Hyperbolic 3-Manifolds” (complete details (and much more) of the construction can be found here)

For connections with Galois representations and automorphic forms:

- (1) Haluk Sengun: “Arithmetic Aspects of Bianchi Groups”
- (2) Haluk Sengun: “Some Applications of Number Theory to 3-Manifolds Theory”

For the torsion homology growth issue:

- (1) Haluk Sengun: “Torsion in the Homology of Arithmetic Kleinian Groups”
- (2) Jean Raimbault: “Torsion Homology of Three-Manifolds” (a very geometric survey)
- (3) Nicolas Bergeron: “Torsion homology growth in arithmetic groups” (for general arithmetic groups)