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Title: Introduction to L^2 -invariants II

In the first week, Lukasz Grabowski introduces L^2 -invariants, with a focus on L^2 -Betti numbers, discussing in particular aspects of the Atiyah conjecture about possible values of these L^2 -Betti numbers, and focusing on the combinatorial aspects of these invariants.

The part II will complement this introduction focusing in particular on:

1. Differential topological: L^2 -invariants defined as invariants of the Laplace-Beltrami operator.
2. Dodziuk's L^2 -Hodge-de Rham theorem: combinatorial and analytic L^2 -invariants coincide
3. finer invariants beyond the dimension of the kernel: L^2 -Betti numbers, Novikov-Shubin invariants, spectral density functions and their dilatation class
4. even finer invariants: L^2 -determinants and L^2 -torsion: definitions, properties, applications, computations.
5. perhaps: a glimpse on the equality of analytic and combinatorial L^2 -torsion
6. perhaps: Computations of L^2 -invariants for locally symmetric spaces: a (more or less difficult) exercise in harmonic analysis
7. if time permits: Analytic proof of the integrality of L^2 -Betti numbers for space with free fundamental group due to Linnell

References

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