

Speaker: Andrei Jaikin

Title: The theory of epic $*$ -regular R -rings and its application to the study of L^2 -Betti numbers

Monday-Thursday (April 23-26, 2018)

10:00-11:15 Lecture

11:15-11:45 Coffee break

11:45-13:00 Lecture

Let K be a subfield of \mathbb{C} . Let F be a free finitely generated group and let $A \in \text{Mat}_{n \times m}(K[F])$ be a matrix over $K[F]$. For each finite F -set X (F acts on the right side) we put

$$\text{rk}_X(A) = \frac{\dim \text{rk}_{\mathbb{C}} \phi_X^A}{|X|},$$

where $\phi_X^A : l^2(X)^n \rightarrow l^2(X)^m$ is an operator induced by right multiplication by A .

Similarly, for each quotient $G = F/N$ of F we can define a von Neumann rank function

$$\text{rk}_G(A) = \dim_G \overline{\text{Im}} \phi_G^A,$$

where $\phi_G^A : l^2(G)^n \rightarrow l^2(G)^m$ is an operator induced by right multiplication by A and \dim_G is the G -dimension on Hilbert G -modules.

The purpose of the course is present the proof of the following theorem.

Theorem. *Let $\{X_i\}$ be a family of F -sets that approximates the group $G = F/N$. Then*

$$\lim_{i \rightarrow \infty} \dim \text{rk}_{X_i}(A) = \text{rk}_G(A).$$

The case $K = \bar{\mathbb{Q}}$ is due to W. Lück [4], J. Dodziuk, P. Linnell, V. Mathai, T. Schick, S. Yates [1] and G. Elek, E. Szabó [2]. The main ideas of the proof of this case is explained in the courses on l^2 -invariants of L. Grabowski and T. Schick during the Introductory School. The proof uses the solution of the so called determinant conjecture in the case of sofic groups. This method does not work for general K .

During the course I will explain the approach, introduced in [3], that uses the theory of epic $*$ -regular R -rings, and allows to prove the theorem for an arbitrary K .

The plan of the course:

- (1) The sofic Lück approximation: the explanation of the problem and the Lück proof in the case $K = \mathbb{Q}$.
- (2) Von Neumann regular and $*$ -regular rings. $*$ -regular closure.
- (3) Sylvester rank functions.
- (4) Epic homomorphisms. Epic von Neumann regular rings and Sylvester rank functions. Epic $*$ -regular R -rings.
- (5) Natural extensions of Sylvester rank functions. The description of the proof of the sofic Lück approximation over arbitrary field of characteristic 0.
- (6) The centralizer of an operator in the space of Hilbert-Schmidt operators.
- (7) The strict eigenvalue property.
- (8) Applications: the Atiyah conjecture, the algebraic eigenvalue conjecture, the center conjecture and the independence conjecture.

REFERENCES

- [1] J. Dodziuk, P. Linnell, V. Mathai, T. Schick, S. Yates, Approximating L^2 -invariants and the Atiyah conjecture., Comm. Pure Appl. Math. 56 (2003), 839–873.
- [2] G. Elek, E. Szabó, Hyperlinearity, essentially free actions and L^2 -invariants. The sofic property, Math. Ann. 332 (2005), 421–441.
- [3] A. Jaikin-Zapirain, The base change in the Atiyah and the Lück approximation conjectures, preprint 2017, <http://verso.mat.uam.es/~andrei.jaikin/preprints/sac.pdf>.
- [4] W. Lück, Approximating L^2 -invariants by their finite-dimensional analogues. Geom. Funct. Anal. 4 (1994), 455–481.