Speaker: Andrei Jaikin

Title: Mod-p methods and the p-gradient

Let G be a group, K a field and A a n by m matrix over the group ring $K[G]$. Let $G = G_1 > G_2 > G_3 \cdots$ be a chain of normal subgroups of G of finite index with trivial intersection. The multiplication on the right side by A induces linear maps

$$\phi_i : \quad K[G/G_i]^n \quad \rightarrow \quad K[G/G_i]^m$$

$$(v_1, \ldots, v_n) \quad \mapsto \quad (v_1, \ldots, v_n)A.$$

We are interested in properties of the sequence $\{\frac{\dim_K \ker \phi_i}{|G:G_i|}\}$. These numbers appear naturally in the study of dimensions of the homology groups $H_p(G_i \backslash X, K)$ where X is G-CW-complex of finite type. In particular, we would like to address the following questions:

(1) Is there the limit $\lim_{i \to \infty} \frac{\dim_K \ker \phi_i}{|G:G_i|}$?
(2) If the limit exists, how does it depend on the chain $\{G_i\}$?
(3) How can we express $\lim_{i \to \infty} \frac{\dim_K \ker \phi_i}{|G:G_i|}$ in terms of G?

It turns out that the answers on these three questions are known if we assume that K is of characteristic 0 [10, 4, 6].

In this course I will present what is known in the case where K is of positive characteristic.

The plan of the course:

(1) The p-gradient. Examples and motivations.
(2) The case of amenable groups.
(3) Virtually pro-p and p-adic analytic sequences.
(4) The case of free groups and posible applications to the Calegari-Emerton conjecture.

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