Fuglede-Kadison Determinants and Sofic Entropy

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Abstract

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Contents

The goal of this mini-course is to sketch the main ideas behind the following results:

1. Ben Hayes' Theorem that expresses the sofic entropy of a principal algebraic action in terms of a Fuglede-Kadison determinant [Hay16],
2. the Li-Thom Theorem that expresses the entropy of a large class of algebraic actions in terms of L^2-torsion [LT14],
3. Ben Hayes' Theorem that expresses the mean dimension of a large class of algebraic actions in terms of a von Neumann dimension, [Hay17b]

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4. the proof of Deninger's conjecture for sofic groups (by Kerr-Li and Hayes) which states that if $f \in M_n(\mathbb{Z}\Gamma) \cap GL_n(\ell^1(\Gamma))$ is not invertible in $M_n(\mathbb{Z}\Gamma)$ then $\det_{L\Gamma}(f) > 1$ [KL13, Hay17a].

The plan of the course:

1. Overview & History
2. Tutorial on Fuglede-Kadison determinants, von Neumann dimension and algebraic actions
3. Tutorial on sofic entropy theory
4. Hayes' Theorem [Hay16]
5. Addition formulas (after Hanfeng Li [Li12])
6. L^2-torsion and the Li-Thom Theorem [LT14]
7. Sofic mean dimension (after Hanfeng Li [Li13])
8. Mean dimension for algebraic actions (after Ben Hayes) [Hay17b]
9. Independence tuples (after Kerr-Li [KL13])
10. Deninger's problem (after Ben Hayes [Hay17a])

References

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