Von Neumann regular rings and $*$-regular rings

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Lecture 1: Basics on von Neumann regular rings

(1) Finitely generated ideals in regular rings
(2) Extensions of regular rings
(3) Matrix rings
(4) Structure of projective modules
(5) Homological characterizations of regular rings

Lecture 2: $*$-regular rings

(1) Proper and positive definite involutions
(2) Left and right projections in $*$-regular rings. The relative inverse of an element
(3) Matrix rings over $*$-regular rings
(4) The main example: the algebra $\mathcal{U}$ of affiliated unbounded operators of a finite von Neumann algebra $\mathcal{A}$
(5) $\mathcal{U}$ is the classical ring of quotients of $\mathcal{A}$
(6) Continuous geometries

Lecture 3: Self-injective regular rings

(1) Unit-regularity and general comparability
(2) Injectivity in the category of modules. Self-injective rings. Injective envelope
(3) Regular right self-injective rings satisfy general comparability
(4) Structure theory for regular, right self-injective rings
(5) Continuous regular rings
(6) The algebra of affiliated operators is self-injective and unit-regular

Lecture 4: Rank functions on regular rings

(1) Definition and first properties of pseudo-rank functions on regular rings
(2) The compact convex set $\mathbb{P}(R)$ of pseudo-rank functions on a regular ring R
(3) Completions of regular rings with respect to a pseudo-rank function
(4) Relationships between N-completeness and self-injectivity for regular rings
(5) Pseudo-rank functions on $*$-regular rings
(6) $\mathcal{U}$ is the rank completion of $\mathcal{A}$

References

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2. Sterling K. Berberian, *The maximal ring of quotients of a finite von Neumann algebra*, Rocky Mountain J. Math. **12** (1982), 149–164.

3. Kenneth R. Goodearl, *von Neumann regular rings*, Monographs and Studies in Mathematics, 4. Pitman (Advanced Publishing Program), Boston, Mass.-London, 1979. Second edition: Robert E. Krieger Publishing Co., Inc., Malabar, FL, 1991.
4. Holger Reich, *Group von Neumann algebras and related algebras*, Ph.D. Dissertation, Götingen, 1999, available at http://www.mi.fu-berlin.de/math/groups/top/members/reich-published.html.