Speaker: Miklos Abert

Title: Groups and graph limits

Abstract. Let Γ be a finitely generated group and $\Gamma_n \leq \Gamma$ be a sequence of subgroups of finite index. Often, Γ arises as the fundamental group of a finite volume manifold or a finite complex M. In this case, Γ_n will give rise to a sequence of finite sheeted covers $M_n = \widetilde{M}/\Gamma_n$ where $\widetilde{M}$ is the universal cover of M. In general, one can visualize Γ_n by fixing a finite generating set S of Γ and then considering the Schreier graphs $\mathrm{Sch}(\Gamma/\Gamma_n, S)$.

We will relate the growth of interesting group invariants, like $d(\Gamma_n)$, the minimal number of generators of Γ_n, Betti numbers over various fields, or the torsion in homology of Γ_n, to combinatorial invariants of the qoutient Schreier graphs, like the eigenvalue distribution and to measured invariants, like the cost or L^2 Betti numbers. Often when one normalizes such an invariant the right way, the limit will be a natural analytic or measured invariant. For instance, when Γ_n forms a chain, we will have

$$\lim_{n\to\infty} \frac{d(\Gamma_n) - 1}{|\Gamma : \Gamma_n|} = \mathrm{cost}(\overline{\Gamma})$$

where $\overline{\Gamma}$ is the profinite completion of Γ with respect to Γ_n and cost is the groupoid cost of a measure preserving action. Similarly, when Γ_n approximates Γ, the growth of Betti numbers over zero characteristic will be equal to the L^2-Betti numbers of Γ.

The approximation property used above comes from Benjamini-Schramm convergence of finite graphs. This is a notion under intense investigation in graph theory in the last 20 years. It also allows one to define sofic groups and prove basic theorems on them. It turns out that in this situation, the convergence can also be expressed by using invariant random subgroups. An invariant random subgroup is a conjugacy invariant probability measure on the space of subgroups.

Course plan:
1) Benjamini-Schramm convergence and spectral measure
2) Invariant random subgroups
3) Rank gradient and cost
4) Putting all together

References

[7Sam] M. Abert, N. Bergeron, I. Biringer, T. Gelander, N. Nikolov, J. Raimbault, and I. Samet, On the growth of L2-invariants for sequences of lattices in Lie groups, Annals of Mathematics 185 (2017), 711–790.

[AGV] M. Abért, Y. Glasner and B. Virág, Kestens theorem for invariant random subgroups, Duke Math. 163 (3), 465-488.

[AbN] M. Abért, T. Gelander and N. Nikolov, Rank, combinatorial cost, and homology torsion growth in higher rank lattices, Duke Math. 166 (15), 2925-2964.

[AbN] M. Abért and N. Nikolov, 'Rank gradient, cost of groups and the rank vs Heegaard genus conjecture', J. Eur. Math. Soc., J Eur Math Soc, 14 (2012), 1657-1677.

[AJN] M. Abért, N. Nikolov and A. Jaikin-Zapirain, 'The rank gradient from a combinatorial viewpoint', Groups Geom. Dyn. 5 (2011), no. 2, 213–230.

[AbW] M. Abért, B. Weiss, Bernoulli actions are weakly contained in any free action, Ergodic theory and dynamical systems 33 (02), 323-333.

[El] G. Elek, The combinatorial cost, Enseign. Math. (2) 53 (2007), no. 3-4, 225–235.

[Far] M. Farber, Geometry of growth: approximation theorems for L^2 invariants, Math. Ann. 311 (1998), no. 2, 335–375.

[Gab] D. Gaboriau, Coût des relations d'équivalence et des groupes. (French) [Cost of equivalence relations and of groups] Invent. Math. 139 (2000), no. 1, 41–98.

[Weiss] B. Weiss, Sofic groups and dynamical systems, Sankhya : The Indian Journal of Statistics 2000, Vol. 62, Ser. A, Pt. 3, pp. 350-359.