Speaker: Miklos Abert

Title: Groups and graph limits

Abstract. Let $\Gamma$ be a finitely generated group and $\Gamma_n \leq \Gamma$ be a sequence of subgroups of finite index. Often, $\Gamma$ arises as the fundamental group of a finite volume manifold or a finite complex $M$. In this case, $\Gamma_n$ will give rise to a sequence of finite sheeted covers $M_n = M/\Gamma_n$ where $M$ is the universal cover of $M$. In general, one can visualize $\Gamma_n$ by fixing a finite generating set $S$ of $\Gamma$ and then considering the Schreier graphs $\text{Sch}(\Gamma/\Gamma_n, S)$.

We will relate the growth of interesting group invariants, like $d(\Gamma_n)$, the minimal number of generators of $\Gamma_n$, Betti numbers over various fields, or the torsion in homology of $\Gamma_n$, to combinatorial invariants of the quotient Schreier graphs, like the eigenvalue distribution and to measured invariants, like the cost or $L^2$ Betti numbers. Often when one normalizes such an invariant the right way, the limit will be a natural analytic or measured invariant. For instance, when $\Gamma_n$ forms a chain, we will have

$$\lim_{n \to \infty} \frac{d(\Gamma_n) - 1}{|\Gamma : \Gamma_n|} = \text{cost}(\Gamma)$$

where $\overline{\Gamma}$ is the profinite completion of $\Gamma$ with respect to $\Gamma_n$ and cost is the groupoid cost of a measure preserving action. Similarly, when $\Gamma_n$ approximates $\Gamma$, the growth of Betti numbers over zero characteristic will be equal to the $L^2$-Betti numbers of $\Gamma$.

The approximation property used above comes from Benjamini-Schramm convergence of finite graphs. This is a notion under intense investigation in graph theory in the last 20 years. It also allows one to define sofic groups and prove basic theorems on them. It turns out that in this situation, the convergence can also be expressed by using invariant random subgroups. An invariant random subgroup is a conjugacy invariant probability measure on the space of subgroups.

Course plan:

1) Benjamini-Schramm convergence and spectral measure
2) Invariant random subgroups
3) Rank gradient and cost
4) Putting all together

References


[AbW] M. Abért, B. Weiss, Bernoulli actions are weakly contained in any free action, Ergodic theory and dynamical systems 33 (02), 323-333.


