

Thematic program “ L^2 -invariants and their analogues in positive characteristic” (ICMAT (Madrid), February 19-June 15, 2018)

1 Scientific description

The thematic program will be devoted to recent developments of the theory of L^2 -invariants and their analogues in positive characteristic. The theory of L^2 -invariants has its origin in a work of M. Atiyah [3], where he proposed an extension of the Atiyah-Singer index theory of elliptic differential operators on compact manifolds to the non-compact case. The modern definition of these invariants is more algebraic and uses the language of CW-complexes (see [7, 18]). The analogue of the first L^2 -Betti number in positive characteristic, the p -gradient, was introduced by M. Lackenby [14] in his study of hyperbolic 3-manifold groups.

In general, L^2 -invariants and their mod p analogues are modeled on classical notions like homology, Betti numbers or Reidemeister torsion for compact spaces and extend these to non-compact spaces with appropriate group actions. There are many applications of L^2 -invariants to problems about groups, manifolds and K-theory. These problems are a priori nothing to do with L^2 -invariants but their solution heavily relies on L^2 -methods. It has already been mentioned the Atiyah-Singer index theory. M. Atiyah also showed that the L^2 -Betti numbers can be used to compute the Euler characteristic of a manifold and this was applied by M. Gromov [10] in order to prove certain cases of the Hopf conjecture about the sign of the Euler characteristic of negatively curved manifolds. W. Lück [19] applied L^2 -invariants in the study of the Baum-Connes conjecture and A. Jaikin [12] in his work on the Hanna-Neumann conjecture.

The study of L^2 -invariants requires input from and is linked to topology, geometry, global analysis, operator theory, ring theory, group theory and K-theory and is of interest for representatives of these fields. A lot of work about L^2 -invariants and their mod p analogues have successfully been done but also a lot of very interesting problems are still open.

The main topics of the thematic program are:

1. **The Atiyah conjecture and its variations.** This conjecture predicts the range of possible values of L^2 -Betti numbers of a given group. This conjecture implies, in particular, the Kaplansky zero-divisor conjecture for group algebras over fields of zero characteristic. Many cases of the conjecture are known, but it is still open, for example, in the case of one-relator groups.

Analogues of the Atiyah conjecture in positive characteristic are also of big interest.

2. **The Lück approximation conjecture.** This conjecture predicts that the function, that calculates the dimension of the kernel of operators corresponding to a fixed matrix over the complex group ring of a free group, defines a continuous function on the space of marked groups. A particular case of special interest is the case of sofic groups. This case is known when the coefficients are algebraic and it is still open when the field is arbitrary.

The Lück approximation in positive characteristic is well understood for amenable groups, but it is widely open in general case.

3. **The entropy of algebraic actions and the Fuglede-Kadison determinant.** An amazing relation ([15, 11]) between these two invariants was discovered for sofic groups. Further understanding of this relation may provide new insights on the problems about the homology growth and L^2 -torsions.

4. **The Calegari-Emerton conjecture.** This conjecture (see [6]) is a particular case of the Lück approximation conjecture in positive characteristic. It predicts that the growth of the dimensions of the mod- p cohomology groups of congruence covers of arithmetic hyperbolic groups is zero. The analogues in characteristic zero is a result of J. Lott and W. Lück.

5. **The Bergeron-Venkatesh problem.** This conjecture (see [4, 5]) predicts the rate of growth of torsion homology in chains of congruence subgroups of an arithmetic groups. In particular, it implies that arithmetic hyperbolic 3-manifolds should have a lot of torsion in their homology. The last result would have important consequences in the theory of Galois representations of the absolute Galois group of \mathbb{Q} .

References

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2 Scientific committee

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3 Local committee

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4 Activities

4.1 Inaugural Colloquium

On Tuesday, February 27th at 12:00, Wolfgang Lück will give the inaugural colloquium for a general audience.

4.2 School on asymptotic invariants of groups

This is a two-week school for graduate students and other young researchers which are interested in, but not necessarily working on the topics of the thematic program. It will be at the beginning of the program (February 26-March, 9, 2018). The level of courses will be introductory. We are planning to have about 4-6 courses per week of 4-6 hours each. We expect to have about 50 participants. The following is a preliminary list of courses and speakers:

First week (February 26-March 2, 2018)

1. **Introduction to L^2 -invariants I** (Lukasz Grabowski)

This course will introduce L^2 -invariants. We are going to have two courses (one each week) and the second will be a continuation of the first one.

2. **Introduction to von Neumann algebras and algebras of affiliated operators.** (Jesse Peterson)

3. **Introduction to measured group theory** (Damien Gaboriau)

4. **Introduction to von Neumann regular and $*$ -regular rings** (Pere Ara)

5. **Mod- p methods and the p -gradient** (Andrei Jaikin)

Second week (March, 5-9, 2018)

6. **Introduction to L^2 -invariants II** (Thomas Schick (TBC))

7. **Groups and graph limits** (Miklos Abert)

8. **Introduction to arithmetic groups and their cohomology** (Steffen Kionke)

9. **Torsion homology of 3-manifolds groups** (Haluk Sengun)

10.

4.3 Advanced courses

During the thematic program we are planning to have 4-5 advanced courses of 8-10 hours each. The following is a preliminary list of courses and speakers:

1. **Sofic entropy and finite automata** (Miklos Abert, March 12-16)
2. **L^2 -Alexander torsion of 3-manifolds** (Stefan Friedl, March 19-23)
3. **Fuglede-Kadison Determinant and Sofic Entropy** (Lewis Bowen, April 16-20)
4. **The theory of epic $*$ -regular rings and its application to the study of L^2 -Betti numbers** (Andrei Jaikin, April 23-27)

4.4 Weekly research seminar

We are planning to have a weekly seminar, where the visitors of the program will present their work.

4.5 Weekly study seminar

We are planning to have a weekly seminar, dedicated to the study of the newest developments in the subject of the program.

4.6 Workshop

At the end of the program (June, 4-8, 2018) we will organize a (Oberwolfach style) workshop. We expect to have about 60 participants.

5 Financial support

We will offer support for short and long visitors of the program (information will appear in due time on the web site of the program).