Maximal functions: boundedness and dimensions

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Starting with the seminal work of Stein in the early eighties, giving dimension independent bounds for the centered maximal function associated to euclidean balls, the dimension dependency of bounds for operators defined by different balls, different measures, and different spaces, has been studied in several works.

We shall discuss some recent developments by different authors in this subject.

Hardy and uncertainty inequalities on stratified Lie groups

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We will consider various Hardy-type inequalities on a stratified Lie group G. In particular, we will show that the operators $f \mapsto |\cdot|^{-\alpha} L^{-\alpha/2} f$, where $|\cdot|$ is a homogeneous norm, $0 < \alpha < Q/p$ and L is a sub-Laplacian, are bounded on the Lebesge space $L^p(G)$. As a consequence, we estimate the norms of these operators precisely enough to be able to differentiate and prove a logarithmic uncertainty inequality. This is a joint work with Michael Cowling and Fulvio Ricci.

BMO spaces for probability measures

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In this talk we consider the problem of finding BMO-type spaces which are suitable for the study of Calderón-Zygmund operators on probability spaces. In particular, we try to characterize spaces containing L^∞ in which these operators are bounded and that are also a valid endpoint for interpolation. Our method is based on previous results by Carbonaro, Mauceri y Meda and can be extended to the setting of operator valued functions. The key element in our scheme is a characterization of L^p as the intersection of two nontrivial quotients of itself, a result of independent interest.

Joint work with Tao Mei and Javier Parcet.

C*-algebras and weak* fixed point property

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Let A be a separable C∗-algebra. We show that the dual $\widetilde{A}$ of A is discrete if and only if the Banach dual A^* of A has the weak* fixed point property. Using this we show that a few more properties of A^* are equivalent to discreteness of $\widetilde{A}$, among them the uniform weak* Kadec-Klee property for A^* and the coincidence of the weak* and the norm topology on the pure states of A (resp. the unit sphere of A^*). This extends results about the special case of the C^*-algebra of alocally compact group and its Fourier-Stieltjes algebra, obtained by Fendler, Lau, and Leinert. In the group case, separability is not needed, in the general case it is an essential assumption.

Elements in the Fourier-Stieltjes algebra vanishing at infinity

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The Fourier algebra $A(G)$ and the Fourier-Stieltjes algebra $B(G)$ are function algebras that occur naturally in harmonic analysis of a locally compact group G. Unless G is compact, $A(G)$ is a proper subalgebra of $B(G)$, since functions in $A(G)$ vanish at infinity while $B(G)$ contains the constant functions. Consider the following question: Does the Fourier algebra $A(G)$ coincide with the subalgebra of $B(G)$ consisting of functions vanishing at infinity?

The talk will cover known results concerning this question. It will also include a theorem giving sufficient conditions for the question to have an affirmative answer.

As an application of the theorem we are able to give new examples of groups G such that $A(G)$ coincides with the subalgebra of $B(G)$ consisting of functions vanishing at infinity.

On some problems in pointwise ergodic theory and additive combinatorics

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Let $\mathbf{P}$ denote the set of prime numbers. We are going to prove $\ell^r(\mathbb{Z})$–boundedness ($r > 1$) of the discrete maximal function

$$M_{\mathbf{P}_h} f(x) = \sup_{N \in \mathbb{N}} \frac{1}{|\mathbf{P}_h \cap [1, N]|} \Big| \sum_{p \in \mathbf{P}_h \cap [1,N]} f(x - W(p)) \Big|, \quad \text{for} \quad x \in \mathbb{Z},$$

where $W : \mathbb{Z} \mapsto \mathbb{Z}$ is a fixed polynomial and

$$\mathbf{P}_h = \{p \in \mathbf{P} : \exists_{n \in \mathbb{N}}\ p = [h(n)]\},$$

for an appropriate function h. As a consequence we obtain related pointwise ergodic theorems along the set $\mathbf{P}_h$.

Finally we show that every subset of $\mathbf{P}_h$ having positive relative upper density contains a nontrivial three–term arithmetic progression. In particular the set of Piatetski–Shapiro primes of fixed type $71/72 < \gamma < 1$, i.e. $\{p \in \mathbf{P} : \exists_{n \in \mathbb{N}}\ p = \lfloor n^{1/\gamma} \rfloor\}$ has this feature. We show this by proving the counterpart of Bourgain–Green's restriction theorem for the set $\mathbf{P}_h$.

Extending a maximal function on compact semisimple Lie groups for singular elements

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Let G be a compact semisimple Lie group with finite centre. We can define a maximal operator $\mathcal{M}_H$ on $\mathcal{C}(G)$ via the $\mathrm{Ad}(G)$-invariant probability measure on the conjugacy class of $\exp(sH)$. Previous work by Cowling, Gunawan and Meaney shows that $\mathcal{M}_H$ is bounded on $L^p(G)$ for $s > 0$,H regular, and $p > 2$. This talk will quickly review these results and then outline the boundedness for $\mathcal{M}_H$ on $L^p(G)$ for singular elements of the Lie algebra.

Some sharp restriction inequalities

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The goal of this talk will be to find the sharp forms and characterize the complex-valued extremizers of the adjoint Fourier restriction inequalities on the sphere:

$$\|\widehat{f\sigma}\|_{L^p(\mathbb{R}^d)} \lesssim \|f\|_{L^q(\mathbb{S}^{d-1},\sigma)},$$

in the cases $(d,p,q) = (d,2k,q)$ with $d,k \in \mathbb{N}$ and $q \in \mathbb{R}^+ \cup \{\infty\}$ satisfying:

(a) $k = 2$, $q \geq 2$ and $3 \leq d \leq 7$;

(b) $k = 2$, $q \geq 4$ and $d \geq 8$;

(c) $k \geq 3$, $q \geq 2k$ and $d \geq 2$.

Tools include a spherical harmonic decomposition, the study of the Cauchy-Pexider functional equation for sumsets of the sphere, and a sharp multilinear weighted restriction inequality with weight related to the k-fold convolution of the surface measure.

This topic builds up on recent results by D. Foschi, and is joint work with E. Carneiro.

Restricted convolution inequalities, multilinear operators and applications

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Multilinear generalized Radon transforms are natural extensions of the linear generalized Radon transforms introduced by Phong and Stein. Estimates on these have already proved useful in establishing results in point configuration problems. In this talk we will introduce new estimates for these operators that are derived from certain convolution type inequalities. We will also discuss applications to multilinear variants of Stein's maximal spherical operators, new Fourier restriction estimates and certain Sobolev trace inequalities.

Harmonic analysis associated with a discrete Laplacian

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We consider the discrete Laplacian

$$\Delta_d f(n) = f(n+1) - 2f(n) + f(n-1), \qquad n \in \mathbb{Z}.$$

By using the heat semigroup, we define and study classical operators of the harmonic analysis associated with Δ_d, namely the fractional Laplacian, maximal heat and Poisson semigroups, square functions, Riesz transforms and conjugate harmonic functions. First, a maximum principle for the fractional Laplacian is obtained. Then, the behavior in weighted $\ell^p(\mathbb{Z})$ spaces, with weights in the Muckenhoupt class, is discussed for the other operators. Besides, it is shown that the Riesz transforms coincide essentially with the so called discrete Hilbert transform defined by D. Hilbert at the beginning of the XX century. We also see that these Riesz transforms are limits of the conjugate harmonic functions.

A characterization of the Ornstein-Uhlenbeck Lipschitz space and estimates for the Ornstein-Uhlenbeck Poisson kernel

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In the n-dimensional Ornstein-Uhlenbeck setting, a Lipschitz space was defined by Gatto and Urbina, in terms of the gradient of the Ornstein-Uhlenbeck Poisson integral of the function. We show that this space can also be described as a Lipschitz space in the ordinary sense, by means of an inequality for the modulus of continuity. The proof is based on several estimates for the Ornstein-Uhlenbeck Poisson kernel and its gradient, also of independent interest.

Semigroups and singular integrals on compact Riemannian manifolds

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The compact Riemannian symmetric spaces of rank one are the basic model manifolds in Riemannian geometry. These are particularly interesting because they admit special global *polar coordinates*, which allow to define Lebesgue mixed-norm spaces $L^p_{\mathrm{rad}}(L^2_{\mathrm{ang}})$ parallel to those of $\mathbb{R}^n$. It is on these spaces where we study estimates for the fractional integral and the Riesz transforms given by the Laplace–Beltrami operator. The key tool is the analysis of the corresponding operators defined through the Poisson semigroup in the context of trigonometric Jacobi polynomials.

Weak type estimates for maximal functions in high dimension

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It was show by S&Stein in early 1980-th that the best constant B_n for weak type estimates of Hard-Littlewood's maximal function centered cubes in $\mathbb{R}^n$, is bounded by $Cn \log n$, with the constant C independent of the dimension. The questions also was raised if B_n has to go to infinity as n goes to infinity. An affirmative answer to this question was given much later by Aldaz, who showed that B_n goes to infinity with n. In a joint work with my PhD student Alexander Iakovlev, we get the more quantative estimate $B_n \geq Cn_{1/4}$.

Cotlar's ergodic theorem along the prime numbers

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Let $(X, \mathcal{B}, \mu, S)$ be a dynamical system. For $f \in L^r(\nu)$ and $N \in \mathbb{N}$ we consider a truncated Hilbert transform

$$\mathcal{H}_N f(x) = \sum_{p \in \pm \mathbb{P}_N} f(S^p x) \frac{\log p}{p},$$

where $\mathbb{P}_N$ is the set of prime numbers inside the interval $[2, N]$. We show that the maximal function $\mathcal{H}_* f(x) = \sup_{N \in \mathbb{N}} |\mathcal{H}_N f(x)|$ is bounded on $L^r(\nu)$, $1 < r < \infty$. Based on that we conclude the existence of the almost everywhere convergence of

$$\lim_{N \to \infty} \mathcal{H}_N f(x).$$

Dimension free L^p estimates for Riesz transforms and H^∞ joint functional calculus

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In 1983 E. M. Stein proved dimension free L^p bounds for classical Riesz transforms on $\mathbb{R}^d$. Since then many authors studied the phenomenon of dimension free estimates for Riesz transforms defined in various contexts. In this talk we present a fairly general scheme for deducing the dimension free L^p boundedness of d-dimensional Riesz transforms from the L^p boundedness of one-dimensional Riesz transforms. The crucial tool we use is an H^∞ joint functional calculus for strongly commuting operators. The scheme is applicable to all Riesz transforms acting on 'product'spaces, e.g.: Riesz transforms connected with (classical) multi-dimensional orthogonal expansions and discrete Riesz transforms on products of groups having polynomial growth.