Generating monotone quantities for the heat equation

Jonathan Bennett, University of Birmingham

j.bennett@bham.ac.uk

The identification of functionals which vary monotonically as their inputs flow according to a given evolution equation is generally considered to be more of an art than a science. Such monotonicity results have many consequences and, in particular, allow for a deeper understanding of a variety of important inequalities in analysis. In this talk we describe a simple framework within which monotone quantities for the heat equation may be "generated", and describe their relevance in harmonic analysis.

Uniform spectral gaps and random matrix products

Emmanuel Breuillard, Université Paris Sud, Orsay

emmanuel.breuillard@math.u-psud.fr

In this talk I will discuss spectral properties of discrete subgroups of Lie groups. Non amenable discrete subgroups of a semisimple Lie group are known to be uniformly non amenable, namely the trivial representation is uniformly isolated in the spectrum of their regular representation. I will explain how to use the theory of random matrix products to extend this to all quasi-regular representations, for which the isotropy group is not Zariski dense. This can be viewed as a quantitative refinement of Borel's density theorem. Applications include explicit lower bounds on the spectral gap of expander graphs arising as finite quotients of these groups, as well as uniform spectral gaps for groups of automorphisms of nilmanifolds.

A multilinear duality principle and applications

Anthony Carbery, University of Edinburgh

A.Carbery@ed.ac.uk

It is joint work with Stefan Valdimarsson.

A sharpened Hausdorff-Young inequality

Michael Christ, University of California, Berkeley

mchrist@berkeley.edu

For any locally compact Abelian group, the Hausdorff-Young inequality states that the Fourier transform maps L^p to L^q, where the two exponents are conjugate and $p \in [1, 2]$. For Euclidean space, the optimal constant in the inequality was found Babenko for q an even integer, and by Beckner for general exponents. Lieb showed that all extremizers are Gaussian functions. This is a uniqueness theorem; these Gaussians form the orbit of a single function under the group of symmetries of the inequality.

We establish a stabler form of uniqueness for $1 < p < 2$: (i) If a function f nearly achieves the optimal constant in the inequality, then f must be close in norm to a Gaussian. (ii) There is a quantitative bound involving the square of the distance to the nearest Gaussian.

The qualitative form (i) can be equivalently formulated as a precompactness theorem in the style of the calculus of variations. Form (ii) is a strengthening of the inequality.

The proof relies on ingredients taken from from additive combinatorics. Central to the reasoning are arithmetic progressions of arbitrarily high rank.

Extension and approximation of functions

Charles Fefferman, Princeton University

cf@math.princeton.edu

This talk summarizes what I've learned, and what I still don't know, about existence and computation of smooth extensions of functions defined on subsets of $\mathbb{R}^n$. Joint work with Arie Israel, Bo'az Klartag and Garving Luli.

Multilinear operators and Erdős-Falconer configuration problems

Allan Greenleaf, University of Rochester

`allan@math.rochester.edu`

Configuration problems of Erdős type concern counting the number of times that geometric quantities or configurations (lengths of line segments, areas of triangles, noncongruent simplices, etc.) occur among the points of a discrete set with a large number, N, of points. Falconer type problems are analogues of Erdős problems in the setting of continuous geometry, where a lower bound on the Hausdorff dimension ensures that a set of configurations has positive measure in the appropriate sense. I will describe some of these problems and recent progress that has been made on them using estimates for multilinear operators and other techniques. This is joint work with various subsets of L. Grafakos, A. Iosevich, B. Liu, M. Mourgoglou and E. Palsson.

Approximation properties for groups and von Neumann algebras

Uffe Haagerup, Københavns Universitet

`haagerup@math.ku.dk`

In the talk I will first give an introduction to weak amenability (M. Cowling and U.H: 1989) and to the weaker property AP (J. Kraus and U.H. 1994) for locally compact groups. Moreover I will discuss the relation of these properties to the group von Neumann algebras for lattices in the groups considered. The rest of the talk will be about two recent joint works with T. de Laat, where we prove that every simple connected Lie group of real rank greater or equal to 2 does not have the AP. More generally we have now shown (work in progress) that a connected Lie group has the AP if and only if all the simple Lie algebras occurring in the Levi decomposition of the Lie algebra of G have real rank at most 1. In particular G has AP if and only if its universal covering group has AP.

On the existence of global solutions of the Euler-Maxwell system

Alexandru Ionescu, Princeton University

aionescu@math.princeton.edu

I will discuss recent work on the construction of global smooth solutions of the Euler-Maxwell system in 2D and 3D. This is joint work with Y. Deng, Y. Guo, and B. Pausader.

Finite configurations in sparse sets

Izabella Łaba, University of British Columbia, Vancouver

ilaba@math.ubc.ca

Let $E \subseteq \mathbb{R}^n$ be a compact set of Hausdorff dimension α close to n. Given $n \times (m - n)$ matrices $B_1, \ldots, B_k$, must E contain non-trivial k-point configurations of the form $\{x + B_1 y, \ldots, x + B_k y\}$? We prove that the answer is affirmative under suitable non-degeneracy conditions on the system of matrices, provided that α is sufficiently close to n and that E supports a probability measure obeying appropriate dimensionality and Fourier decay conditions. In particular, our results cover geometric configurations.such as parallelograms in $\mathbb{R}^n$ and isosceles right triangles in $\mathbb{R}^2$). This can be viewed as a multidimensional analogue of a result of Laba and Pramanik on 3-term arithmetic progressions in Salem sets on the line. (Joint work with Vincent Chan and Malabika Pramanik.)

The two weight inequality for the Cauchy transform for $\mathbb{R}$ to $\mathbb{C}$

Michael T. Lacey, Georgia Institute of Technology

lacey@math.gatech.edu

We characterize those pairs of weights, on on the real line, and the other on the complex plane, such that the Cauchy transform is bounded from L^2 of the first weight, to L^2 of the second. The characterization is in terms of a joint Poisson A_2 condition, and a suite of testing inequalities. This verifies a conjecture of Nazarov-Treil-Volberg.

This theorem is well-connected to questions in operator theory and analytic functions spaces. Applications include a question of Donald Sarason on the composition of Toeplitz operators, and another of William Cohn, on the characterization of Carleson measures for model spaces.

Joint work with Sawyer, C.-Y. Shen, Uriate-Tuero and Wick.

Sharp square function estimates for the Bochner-Riesz means

Sanghyuk Lee, Seoul National University

shklee@snu.ac.kr

We consider the square functions associated with the Bochner-Riesz mean which is known as Stein's square function and improve the previously known range of sharp estimates in higher dimensions.

The pointwise convergence of the Fourier Series - the lacunary case. On a conjecture of Konyagin.

Victor Lie, Purdue University

vlie@math.purdue.edu

In his 2006 ICM invited address, Konyagin mentioned the following conjecture: if $S_n f$ stands for the n-th partial Fourier sum of f and $\{n_j\}_j \subset \mathbb{N}$ is a lacunary sequence, then $S_{n_j} f$ is a.e. pointwise convergent for any $f \in L \log \log L$. In this talk we will present the resolution of the above conjecture.

Generalized BMO spaces on noncompact Riemannian manifolds

Giancarlo Mauceri, Università degli Studi di Genova

`mauceri@dima.unige.it`

Let M be a complete connected noncompact Riemannian manifold with bounded geometry and spectral gap. In previous papers with S. Meda and M. Vallarino we introduced on M a family of atomic Hardy spaces $X^k(M)$, $k \in \mathbb{N}$, to obtain endpoint estimates for singular integrals on M, such as Riesz transforms and spectral multipliers of the Laplacian. In this talk I shall present a joint work with Meda and Vallarino, in which we describe the dual space $Y^k(M)$ of $X^k(M)$, as the class of all locally square integrable functions satisfying suitable BMO-like conditions, where the role of the constants is played by the space of global k-quasi-harmonic functions. Furthermore we prove that $Y^k(M)$ is also the dual of the space $X_{\mathrm{fin}}^k(M)$ of finite linear combination of X^k-atoms. As a consequence, if Z is a Banach space and T is a Z-valued linear operator defined on $X_{\mathrm{fin}}^k(M)$, then T extends to a bounded operator from $X^k(M)$ to Z if and only if it is uniformly bounded on X^k-atoms. To obtain these results we prove the global solvability of the generalized Poisson equation $\mathcal{L}^k u = f$ with $f \in L^2_{\mathrm{loc}}(M)$ and we study some properties of generalized Bergman spaces of harmonic functions on geodesic balls.

Hardy type spaces on noncompact manifolds

Stefano Meda, Università di Milano-Bicocca

stefano.meda@unimib.it

I shall survey some comparatively recent results concerning Hardy type spaces on noncompact Riemannian manifolds.

Then I shall focus on a certain class of Riemannian manifolds M having exponential volume growth and containing all Riemannian symmetric spaces of the noncompact type. I shall define the Hardy-type space $X^1(M)$ and show that it plays in Harmonic Analysis on M much the same role as the classical Hardy space $H^1(\mathbb{R}^n)$ plays in Euclidean Harmonic Analysis. In particular, if $1 < p < 2$, then $L^p(M)$ is an interpolation space between $X^1(M)$ and $L^2(M)$. Furthermore some relevant operators, such as spectral multipliers of Mihlin–Hörmander type of the Laplace–Beltrami operator $\mathcal{L}$, and the Riesz transform $\nabla \mathcal{L}^{-1/2}$ are bounded from $X^1(M)$ to $L^1(M)$.

The space $X^1(M)$ is an atomic space. An atom in $X^1(M)$ is supported in a "small" geodesic ball B in M, satisfies a standard size estimate and a suitable infinite dimensional cancellation condition, which will be discussed in detail.

This is joint work with G. Mauceri and M. Vallarino.

Spectral multipliers for sub-Laplacians: recent progress

Detlef Müller, Christian-Albrechts-Universität zu Kiel

mueller@math.uni-kiel.de

For large classes of full Laplacians, such as Laplace–Beltrami operators on compact Riemannian manifolds, sharp spectral multiplier theorems of Mihlin–Hörmander type are well-known. In contrast, for subelliptic Laplacians, such as Grushin operators or sub-Laplacians on Lie groups repectively homogeneous spaces, sharp results are known only in a few cases. This includes the Heisenberg group, but also some compact situtations to which Michael Cowling has contributed, like the group $SU(2)$ and the unit sphere in $2n$-dimensional complex space. In the talk, I shall report on recent results and new methods developed in joint work with Alessio Martini.

Iterated Fourier series

Camil Muscalu, Cornell University

camil@math.cornell.edu

Products of Fourier series split naturally as multiple sums of "iterated Fourier series". Their almost everywhere convergence, turns out to be closely related to problems of "physical reality". The goal of the talk is to describe all of these.

Sharp L^p estimates for discrete second order Riesz transforms

Stefanie Petermichl (joint with K. Domelevo), UPS Toulouse

stefanie.petermichl@gmail.com

We find exact L^p norms of second order Riesz transforms on $\mathbb{Z}^n$. The best constant is the same as in the continuous case: $\max\{p-1, 1/(p-1)\}$. We also treat the case of cyclic groups. The corresponding classical estimate in euclidean space is due to Fedor Nazarov and Alexander Volberg and dates to 2000. We discuss some of the artifacts encountered in discrete spaces, among them a negative (and positive) result by Francoise Piquard.

Quantum expanders and tensor products of operator algebras

Gilles Pisier, Texas A&M University

pisier@math.tamu.edu

The talk will explain the connection of quantum expanders with the non-separability of the set of finite dimensional (actually 3-dimensional) operator spaces which goes back to a 1995 paper with Marius Junge, and several more recent "quantitative" refinements. In particular, we will show a recent joint result with N. Ozawa: For any pair M, N of non-nuclear von Neumann algebras there is a continuum of distinct C*-norms on the algebraic tensor product $M \otimes N$.

Multi-norm singular integrals

Fulvio Ricci, Scuola Normale Superiore, Pisa

fricci@sns.it

Composition of Calderón-Zygmund convolution operators on $\mathbb{R}^n$ with different homogeneities gives rise to new types of operators which are pseudolocal and bounded on L^p for $1 < p < \infty$, but fall outside of the C-Z theory. We introduce classes of kernels adapted to given families of homogeneous norms (with different homogeneities) which are closed under convolution and include C-Z kernels with different homogeneities.

This is joint work with A. Nagel, E. Stein and S. Wainger.

L^p-resolvent estimates for compact Riemannian manifolds and hyperbolic space

Christopher Sogge, Johns Hopkins University

sogge@jhu.edu

I shall discuss joint work with Jean Bourgain, Peng Shao and Xiaohua Yao. We are able to obtain necessary and sufficient conditions for favorable resolvent estimates in terms of improved L^p estimates for shrinking spectral projection operators. Using them we are able to obtain improved resolvent estimates for manifolds of nonpositive curvature and also show that the earlier known results for the sphere of Dos Santos Ferreira, Kenig and Salvo cannot be improved.

I shall also go over recent joint work with Shanlin Huang where we prove resolvent estimates for simply connect manifolds of constant curvature which imply the earlier ones for Euclidean space of Kenig, Ruiz and myself. In the case of hyperbolic space, the key ingredient in the proof is a natural variant of the Stein-Tomas restriction theorem for $\mathbb{H}^n$.

Hilbert's fifth problem and approximate groups

Terence Tao, University of California, Los Angeles

tao@math.ucla.edu

Approximate groups are, roughly speaking, finite subsets of groups that are approximately closed under the group operations, such as the discrete interval $\{-N, \ldots, N\}$ in the integers. Originally studied in arithmetic combinatorics, they also make an appearance in geometric group theory and in the theory of expansion in Cayley graphs.

Hilbert's fifth problem asked for a topological description of Lie groups, and in particular whether any topological group that was a continuous (but not necessarily smooth) manifold was automatically a Lie group. This problem was famously solved in the affirmative by Montgomery-Zippin and Gleason in the 1950s.

These two mathematical topics initially seem unrelated, but there is a remarkable correspondence principle (first implicitly used by Gromov, and later developed by Hrushovski and Breuillard, Green, and myself) that connects the combinatorics of approximate groups to problems in topological group theory such as Hilbert's fifth problem. This correspondence has led to recent advances both in the understanding of approximate groups and in Hilbert's fifth problem, leading in particular to a classification theorem for approximate groups, which in turn has led to refinements of Gromov's theorem on groups of polynomial growth that have applications to the study of the topology of manifolds. We will survey these interconnected topics in this talk.

L^2-Betti numbers for locally compact groups

Alain Valette, Université de Neuchâtel

alain.valette@unine.ch

L^2-Betti numbers were defined for discrete groups by Atiyah (1976) and Cheeger- Gromov (1986), for measurable equivalence relations by Gaboriau (2002), and for arbitrary unimodular locally compact groups by Petersen (2012) and Kyed-Petersen-Vaes (2013). In joint work with Henrik Petersen, we give a formula for computing the n-th L^2-Betti number of a type I, unimodular, locally compact group G: it is given as the integral over the dual $\hat{G}$, with respect to Plancherel measure, of the usual dimension of the reduced n-cohomology of G with coefficients in an irreducible representation. Since the latter has been much studied, we are able to compute the L^2-Betti numbers of real or p-adic semi-simple Lie groups, and automorphisms groups of locally finite trees that act transitively on the boundary.

Restriction theorems for some surfaces of finite type

Ana Vargas, Universidad Autónoma de Madrid

ana.vargas@uam.es

This is a joint work with Detlef Müller and Stefan Buschenhenke. We prove a bilinear restriction theorem for some model surfaces in R^3, with zero curvature but of finite type. As a consequence we obtain new restriction estimates for those surfaces. These theorems are the analogs of Wolff and Tao's theorems for cones and paraboloids.

An operator-valued exponential sum

James Wright, University of Edinburgh

J.R.Wright@ed.ac.uk

As a model for degenerate Fourier integral operators, Phong and Stein considered oscillatory integral operatos with respect to a real binary form. They establish sharp uniform L^2 bounds for these operators. We prove analogous uniform bounds for exponential sums with a general integer binary form as phase. Such estimates can be viewed as a robust version/solution of the Igusa conjectures from number theory for binary forms.