

DENSITY THEOREMS FOR EUCLIDEAN POINT CONFIGURATIONS

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Euclidean density theorems seek for dilated, rotated, and translated copies of a fixed finite point configuration within a “large” measurable subset of the Euclidean space of appropriate dimension. Of particular interest are results that identify the configuration dilated by all sufficiently large scales. Since the seminal paper by Bourgain, which handled configurations forming vertices of a non-degenerate simplex, the proof strategy has always been a certain (explicit or implicit) decomposition of the counting form into structured, error, and uniform pieces, much in the spirit of regularity lemmas from arithmetic combinatorics and graph theory. However, a significant extra input from harmonic analysis is also needed in the proofs, often from the subfields of multilinear singular and oscillatory integrals. We will present several recent results on this topic, discussing: configurations standing in an arithmetic progression, three point corners, vertices of boxes with three-term progressions “attached”, configurations coming from anisotropic dilations, and configurations in sets of density close to 1, and leaving a few open problems. We will also always try to emphasize singular integral operators relevant to the discussed proof. The talk will be based on several papers coauthored with P. Durcik, K. Falconer, L. Rimanic, and A. Yavicoli.