Modified Equations for Variational Integrators

Mats Vermeeren

Technische Universität Berlin

Discretizations of differential equations are often studied through their modified equation. This is a differential equation, usually obtained as a power series in the step size, with a solution that exactly interpolates the discretization. It is well-known that if a symplectic integrator is applied to a Hamiltonian system, then its modified equation is again Hamiltonian. In this talk we discuss this property from the Lagrangian side. We present a technique to construct a Lagrangian for the modified equation directly from the discrete Lagrangian defining the variational integrator. As an example, we apply this method to several integrators for the Kepler problem, and use the results to study the spurious orbital precession that appears in the numerical solutions.

See [1, Chapter IX], [2], and the references therein for the general theory of modified equations. See [3] and [4] for the case of variational integrators.

References

[1] Ernst Hairer, Christian Lubich, and Gerhard Wanner. Geometric numerical integration: structure-preserving algorithms for ordinary differential equations. Springer, 2006.

[2] P C Moan. On modified equations for discretizations of ODEs. Journal of Physics A: Mathematical and General, 39(19):5545, 2006.

[3] Marcel Oliver and Sergiy Vasylkevych. A new construction of modified equations for variational integrators. Preprint, 2012.

[4] Mats Vermeeren. Modified equations for variational integrators. In preparation, 2015.