Bäcklund transformations for Lie-Poisson integrators

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We consider a dynamical system on the dual of a Lie algebra. On the dual of that algebra there is another linear Poisson structure. This system is integrable for one of the Poisson structures because it admits a Lax representation.

Applying the discrete variational principles Lie-Poisson integrators known in the literature are obtained, which preserve all the Casimir functions of the system. In the 19th century Bäcklund transformations were introduced in the area of partial differential equations as transformations that maps solutions to solutions. As reviewed in [1] Bäcklund transformations satisfy some specific properties such as commutativity. We geometrically define Bäcklund transformations associated with the Lie-Poisson integrators under some invariance assumptions. The existence of an invariant scalar product that identifies the Lie algebra and the dual of the Lie algebra allows to establish the connection with the results proved in [1] on the Lie algebra. We will make clear the constructions by looking at the Toda Lattice [2].

References:

[1] Yu.B. Suris. *The Problem of Integrable Discretization: Hamiltonian Approach.* Progress in Mathematics, Vol. 219. Basel: Birkhäuser, 2003.

[2] W. W. Symes. The QR algorithm and scaterring for the finite nonperiodic Toda lattice. *Physica 4D*(1982), 275–281.