Lie bracket flows and post-Lie algebra

Kurusch Ebrahimi-Fard
Instituto de Ciencias Matemáticas
Consejo Superior de Investigaciones Científicas
kurusch@icmat.es

Post-Lie algebras consist of a vector space V equipped with two Lie brackets $[\cdot,\cdot]$ and $[\![\cdot,\cdot]\!]$ as well as a non-commutative and non-associative product $\triangleright : V \otimes V \to V$, such that the following identity holds

$$[\![x,y]\!] = x \triangleright y - y \triangleright x + [x,y].$$

An example is given in terms of a Lie algebra g together with the solution $\pi_- \in \mathrm{End}(g)$ of the modified classical Yang–Baxter equation

$$[\pi_-(x),\pi_-(y)] + \pi_-([x,y]) = \pi_-\big([\pi_-(x),y] + [x,\pi_-(y)]\big),$$

and the corresponding double Lie bracket

$$[x,y]_{\pi_-} := [\pi_-(x),y] + [x,\pi_-(y)] - [x,y].$$

Such maps are called R-matrices, and they are intimately related to solutions of Lie bracket flow equations in terms of a factorization problem in the Lie group G corresponding to g. We will study this R-matrix setting from the point of view of the Lie enveloping algebra of a post-Lie algebra and its Hopf algebra structure. Explicit solutions of the aforementioned Lie bracket flow equations are given in terms of the solution of a post-Lie Magnus-type differential equation. This talk is based on joint work with Alexander Lundervold, Igor Mencattini and Hans Munthe-Kaas.

References:

F. Casas, A. Iserles, *Explicit Magnus expansions for nonlinear equations*, J. Phys. A: Math. Gen., **39**, 5445 (2006).

K. Ebrahimi-Fard, A. Lundervold, H. Munthe-Kaas, *On the Lie enveloping algebra of a post-Lie algebra*, to appear in J. Lie Theory, April 2014.

K. Ebrahimi-Fard, L. Guo, D. Manchon, *Birkhoff type decompositions and the Baker–Campbell–Hausdorff recursion*, Commun. Math. Phys. **267**, 821 (2006).

L. E. Faybusovich, *QR-type factorizations, the Yang–Baxter equation and an eigenvalue problem of control theory*, Linear Algebra Appl. **122-124**, 943 (1989).

N. Yu. Reshetikhin, M. A. Semenov-Tian-Shansly, *Quantum R-matrices and factorization problems*, JGP **5**, no. 4, 533 (1988).

M. A. Semenov-Tian-Shansky, *What is a classical r-matrix?*, Funct. Ana. Appl. **17**, 254 (1983).