A GEOMETRIC INTEGRATOR FOR OPTIMAL CONTROL PROBLEMS

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In this talk we will explore an approach to the geometric integration of optimal trajectories of mechanical systems [1]. Given a mechanical system with a (regular) Lagrangian function $L\colon TQ \to \mathbb{R}$, its controlled Euler–Lagrange equations are, in local coordinates,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^A} - \frac{\partial L}{\partial q^A} = u_A,$$

where $u = (u_A) \in U \subset \mathbb{R}^n$ is an open subset of $\mathbb{R}^n$, the set of control parameters. Given a cost function $C\colon TQ \times U \to \mathbb{R}$, we seek to minimize $\int_{t_0}^{t_f} C(q^A, \dot{q}^A, u_A)dt$. We can see this problem as a Lagrangian system where the second-order Lagrangian $\widetilde{L}\colon T^{(2)}Q \to \mathbb{R}$ is given by

$$\widetilde{L}(q^A, \dot{q}^A, \ddot{q}^A) = C\left(q^A, \dot{q}^A, \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^A} - \frac{\partial L}{\partial q^A}\right)$$

(see for example [2]).

In previous approaches (see for instance [3, 4, 5]), the theory of discrete variational mechanics for second-order systems was derived using a discrete Lagrangian $L_d\colon Q \times Q \times Q \to \mathbb{R}$, where 3 points are used to approximate the positions, velocities and accelerations. Instead of this, we will use a different kind of discrete Lagrangian $L_d\colon TQ \times TQ \to \mathbb{R}$ and explain some of the advantages.

The existence and regularity of the exact discrete Lagrangian will be discussed, as well as the notions of discrete Legendre transforms.

Some concrete examples and simulations will be shown, corresponding to the optimal control of fully actuated systems.

References

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