Lie-Poisson integrators

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In this review talk we will discuss the numerical integration of hamiltonian systems on Poisson manifolds, in particular, in the case, when the Poisson structure is induced by a Lie algebra, that is, it is a Lie-Poisson structure.

A Hamiltonian system on a Poisson manifold (M, Π) is a smooth manifold M equipped with a bivector field Π, with the property that the bracket on $C^\infty(M)$

$$\{f, g\} := \Pi(df, dg), \qquad f, g \in C^\infty(M)$$

is a Lie bracket and, in particular it satisfies the Jacobi identity. For any $f \in C^\infty(M)$ the Hamiltonian vector field is defined by

$$X_f(g) = \{g, f\}$$

The Hamiltonian vector fields X_f generate an integrable generalized distribution on M and the leaves of this foliation are symplectic. The flow of any hamiltonian field preserves the Poisson structure, it fixes each leaf and the energy is a first integral.

We are interested in characterizing numerical methods preserving some of these fundamental properties of the hamiltonian flow on Poisson manifolds.

During the talk, we will discuss the difficulties of deriving these Poisson methods using standard techniques and we will present some modern approaches to the problem.

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