GEOMETRIC NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS IN APPLICATIONS OF COMPUTER ANIMATION AND PORT-HAMILTONIAN SYSTEMS

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Brainstorming Workshop on New Developments in Discrete Mechanics, Geometric Integration and Lie-Butcher Series
25th-28th May 2015, ICMAT, Madrid

The topic of this talk is the use of geometric numerical integration and Lie groups in two particular areas of application. The first application is in problems of computer animation. The second is in the structure preserving discretization of port-Hamiltonian systems. The relevant numerical techniques are mostly, but not exclusively, energy-preserving resp. energy-dissipative numerical methods for ODEs and PDEs on Lie groups.

1. Analysis of shapes on Lie groups and their advantages in computer animation

Motion of virtual characters in video games is usually represented using a skeletal animation. The underlying skeleton consists of bones connected by joints. Animation of virtual characters relies on collections of data obtained by recording the movements of actors. The data consists of curves tracking the positions of the bones throughout the motion, and these curves can be processed by mathematical methods to produce new motions. In practice, the data consists of curves in $SO(3)^N$, where N is the number of bones in the skeleton. In the existing literature these curves are represented using Euler angles and neglecting the underlying Lie group structure, [10], [2]. We here report on the results we obtained by appropriately including the underlying geometric structure in the mathematical models and their numerical discretizations. The intrinsic geometric formulation is robust and works very well in problems of motion blending and curve closing, where earlier the same performance could be obtained only by using ad hoc strategies, e.g. keeping track of carefully chosen feature points along the curves.

We will briefly discuss how techniques from shape analysis on Lie groups can be successfully applied to computer animation treating character animations as points in an infinite dimensional manifold. This manifold is in fact an infinite dimensional Lie group where we

Joint work with M. Eslitzbichler, E.H. Høiseth and N. Ramzina.

are interested in computing distances and geodesics. As expected by smimilar experience in the case of shapes on Euclidean spaces [1], the choice of metrics on these infinite dimensional manifolds are a trade off between ease of computation and the needs dictated by the application at hand. In particular we will discuss elastic metrics and their relation to the L_2 norms for curves on the Lie algebra. Furthermore the extension to higher order Sobolev type norms will be considered.

Working with curves of animation data, we are interested in particular in two mathematical problems: computing geodesics on the infinite dimensional Lie group; and projecting open curves onto the submanifold of closed curves. We will discuss the modeling of these problems by designing appropriate gradient flows. We will finally address the numerical discretization of these PDEs and show the advantages of the proposed approach in animation of character motion.

2. Structure preserving methods for port-Hamiltonian systems

The second part of the talk is devoted to port-Hamiltonian systems. These are a generalization of Hamiltonian systems allowing to include inputs and outputs. Port-Hamiltonian systems offer a simple paradigm for modelling complex physical systems by the energy-consistent interconnection of a (possibly large) number of simple subsystems. This approach can also be viewed as a technique for control design [12, 11, 9]. The structure-preserving (in particular passivity-preserving) integration of this generalisation of Hamiltonian systems is of interest both from a theoretical perspective and in engineering applications.

We will present a number of motivational examples, some simple ones, some of historical interest and one example arising in the maneuvering of vessels for marine operations [6]. We present some preliminary results in the structure preserving discretization of port-Hamiltonian systems. We analyse these methods, focusing in particular on discrete energy-preserving and passivity-preserving interconnection of simpler systems.

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