On some identities involving exponentials

Fernando Casas[a]

[a] Departament de Matemàtiques and IMAC, Universitat Jaume I, 12071-Castellón, Spain. E-mail address: Fernando.Casas@uji.es

The notion of the exponential of a matrix is usually introduced in elementary textbooks on ordinary differential equations when solving a constant coefficients linear system. One can also find there some of its properties and in particular one that *does not* hold unless the involved matrices commute.
Several problems arise indeed from this fundamental issue, and it is our purpose in this talk to review some of them, namely: (1) is it possible to write the product of two exponential matrices as the exponential of a matrix? (ii) is it possible to "disentangle" the exponential of a sum of two matrices? (iii) how to write the solution of a time-dependent linear differential system as the exponential of a matrix?
In our journey, we will touch upon many different fields of mathematics and physics, ranging from the theory of Lie groups and Lie algebras and the numerical analysis of differential equations to quantum mechanics, statistical mechanics, many-body theory, and q-analysis in quantum groups.
We will try to keep the (mathematical) level of the talk deliberately low for more easily grasping the nature of the issues involved. In any case, references will be provided for a more advanced treatment.

References

[1] A. Bonfiglioli and R. Fulci, *Topics in Noncommutative Algebra*, Lecture Notes in Mathematics 2034. Springer, 2012.

[2] C. Reutenauer, *Free Lie Algebras*. Oxford University Press, 1993.

[3] W. Magnus, On the exponential solution of differential equations for a linear operator. *Commun. Pure Appl. Math.* **VII** (1954), 649-673.

[4] W. Magnus, A. Karrass and D. Solitar, *Combinatorial Group Theory*. Dover, 2004.