Infinite-dimensional Lie groups in numerical analysis.

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Many phenomena in numerical analysis can be studied by expanding a function in series over some index set. The combinatorial formulas arising from composing such series can in some cases be formulated via the co-product in a Hopf algebra, with the interesting series becoming character groups with values in the real or complex numbers. In two recent papers [1,2], we have explored the topological geometric aspects of these groups, specifically showing if the Hopf algebra is graded and connected, the character group becomes a locally convex Lie group modelled on an infinite dimensional case. We also show the regularity of these Lie groups, and that the Lie group exponential is a global analytic diffeomorphism.

The most prominent character group arising in numerical analysis is the Butcher group, which is studied in particular in [1]. The Lie group structure on the Butcher group fits well with some phenomena studied in the literature on B-series.

References

[1] G. Bogfjellmo and A. Schmeding. The Lie group structure of the Butcher group. http://arxiv.org/abs/1410.4761
[2] G. Bogfjellmo, R. Dahmen and A. Schmeding. Character groups of Hopf algebras as infinite-dimensional Lie groups. http://arxiv.org/abs/1501.05221