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EDITORIAL

Consequences that exceed the initial intention

Image: ICMAT

Alberto Enciso is codirector of the ICMAT Scientific Committee.

One of my favourite stories is about the fictitious village of Agloe in New York State (USA). In the 1930s, the cartographers Otto G. Lindberg and Ernest Alpers decided to situate a place bearing this name (an anagram formed from their initials) on the road map they were making in order to serve as a copyright trap to catch plagiarists. In later years, when other maps also showed a village called Agloe, located at a dirt-road intersection that Lindberg and Alpers had chosen at random, these two cartographers had no hesitation in suing others for copyright infringement. Their cases, however, met only with failure; perhaps because of their dismay at not finding the village on the road map, people had built a village themselves at the exact location of Agloe, together with a gas station and four farms. This story shows that not only a map is not a territory, but also it may even lead to the creation of one, the consequences of which may far exceed the initial intention.

A much more recent and well-known example is the case of the Covid-19 pandemic and its impact on our lives, our jobs and our research work. The theoretical nature of mathematical research has enabled us to pursue our endeavours with fewer delays and difficulties than in other fields of science that require a greater degree of experimentation. However, the way in which we conduct work has also changed drastically in this period of time. The discipline of mathematics often involves discussions, blackboard work and travel, but during the last two years there has been far less travel, less use of chalk and less face-to-face interaction. Of course, this is only the tip of the iceberg. The elephant in the room consists of mental health problems, mainly concerning anxiety and depression, which constitute a well-documented occupational hazard in the world of research that has only grown worse with the increase in uncertainty and isolation as a result of the pandemic and the feeling of insecurity caused by its recurring emergence. It is important to point out that these circumstances have particularly affected young people, whether or not they happen to be mathematicians; a collective that has frequently been criticized in the media for flagrantly irresponsible social behaviour, even though the overwhelming majority have conducted themselves in an irreproachable way in highly adverse situations.

Since the beginning of the pandemic, the mathematical community has sought ways of mitigating the difficulties that have arisen. Many of us have used digital tablets rather than chalk, a change that has lent an almost epistolary nature to our collaborations, while our discussions have mainly been carried out using Zoom, Teams, Skype and Conecta. This has entailed the creation of new physical and mental spaces for working via video link and for face-to-face interaction without going to the office or travelling by air, as well as adapting timetables and work habits to a series of complex and uncertain conditions. Without adopting such measures, the impact of Covid-19 on our work would have been far worse.

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After almost two years of the pandemic, it is very good news that –with caution, mask-wearing, vaccination and social distancing– we can gradually return to in-person activity and enjoy direct personal interaction once again. Many events and conferences (most of the seminars and colloquia organized by ICMAT, among others) are already being prepared both in-person and online. While we still do not know how many chapters are left before we finish the story of this pandemic, this gradual recovery of person-to-person activity promises to provide a full return to something like the normality to which we were all accustomed.

Since every crisis also presents us with the opportunity to emerge even stronger than before, we should strive to ensure that our work as mathematicians in this new normality is also smoother and more productive than before. As in the case of the fictional village of Agloe, the work tools and methodologies introduced as temporary measures to palliate the effect that the pandemic has had on our work should go much further than that for which they were initially intended. Far from disappearing completely as we return to normal activity, such measures should act as a guide and a reference for the development of more efficient and sustainable ways of working, as well as facilitating a more equitable balance between work and personal life. Both our theorems and our mental health will be much better off as a result.

Alberto Enciso is *profesor de investigación* at the **Consejo Superior de Investigaciones Científicas** and a member of the **ICMAT.**

Image: ICMAT

Concha Bielza gave a virtual and in-person colloquium in October.

REPORT: Science seeks women researchers

The celebration of the 2nd Meeting on Gender Equality of the Alliance between Severo Ochoa centres and María de Maetzu units (SOMMa), held in May 2021 and organized by the ICMAT Equality Commission, left a feeling of some concern in its wake. Despite all the efforts made, the data indicate that science is still far from achieving its goal of equality between men and women. The participants in the meeting state that far more understanding, awareness and training are required to encourage girls to study mathematics and provide support for women already engaged in mathematical research. It is absolutely essential in that regard to bring about a structural transformation, because without equality there can be no excellence.

Elvira del Pozo

Less than one in every three researchers worldwide is a woman, and only three percent of all the Nobel prizes awarded in science have been to women, as stated in a report issued by UNESCO. "This under-representation is mainly found in those scientific fields in which mathematics is largely involved: geosciences, engineering, economics, mathematics itself, computational and physical sciences," says Donna Ginther, a distinguished professor and director of the University of Kansas Institute of Policy and Social Research (USA). She was the main speaker at the 2nd Meeting on Gender Equality of the SOMMa centres and units of excellence, organized in May by the ICMAT Equality Commission.

The gender gap in STEM studies starts at a very early age, far before the university stage, and is closely related to the classes in mathematics given in high school, says Ginther. In her opinion, "girls are so convinced that they are no good at mathematics that they don't even bother to try. It's not a question of lack of aptitude, but rather of stereotyping." This expert believes that a further obstacle is that, quite unconsciously, girls do not want to compete with boys.

Ginther pointed to studies that reflect how teachers of mathematics at a primary level inculcate inequality between boys and girls by tending to encourage the former more than the latter. If, furthermore, a greater gender inequality exists in the personal circumstances of girl pupils, the situation can only become worse. The message that these pupils receive is that "mathematics is not for girls." The upshot is that when girls are faced with the choice of what science subjects to study in high school, they mainly opt for biology rather than physics or mathematics. This closes doors for access to certain degree courses at university, which in turn reduces the number of young women engaged in those STEM studies involving a larger measure of mathematics.

Image: ICMAT

Donna Ginther, director of the Institute of Policy and Social Research, Kansas University (US), gave the conference "Women and STEM: Are Differences in Education and Career due to Stereotypes, Interest or Family?".

Ginther also pointed out that girls (and also minorities) achieve better results in mathematics when they are constantly reassured from the outset that it is a discipline that anyone can learn, and not simply a matter of whether one understands it or not. Improved results are also obtained if art is included in mathematical problems and if no penalties are applied when pupils get the wrong solution. Furthermore, says this expert in scientific labour markets and gender inequality in the academic context, it is "important" for girls to be made aware of women who are successful in STEM fields of research.

Adult difficulties

In addition to these stereotypes forged during the years of education, a career in science does not appear to be particularly attractive to women. "The high level of dedication and the lack of flexibility in working hours are associated with the scant presence of women," Ginther concluded.

In Spain, the proportion of women researchers on the staff of the Consejo Superior de Investigaciones Científicas (CSIC) has hardly changed over the last decade, according to the 2021 Report on Women Researchers drawn up by the Women and Science Commission of this institution. Furthermore, in 2020 the percentage of women with post-doctoral contracts has been the lowest (50.5 %) in the last 16 years, while the PhD theses defended by women have fallen to 45.97 % (in the previous study this figure was 51.6 %).

This trend is also reflected in the Women in Figures statistical series drawn up by the Ministry of Science and Innovation Women and Science Commission. In the two years between 2015 and 2017, female university students who went on to do research dropped by seven points.

"The data show that things are getting worse," said Rosina López-Alonso, the vice-president of Organization and Institutional Relations of the CSIC, who participated in the SOMMa Meeting on Gender Equality. "Although many women graduated in STEM subjects, those who set out on a research career have diminished, as have those who ascend to higher scientific positions and those who defend a PhD thesis." She also pointed out that fewer women respond to public calls, both for research posts and for grants.

She went on to say that "it seems that difficulty in balancing work and private life puts women off from taking up a career in research." In her opinion, such a career is highly demanding in terms of mobility, given the need to do international stays, as well as in working hours and the effort required. She also remarks that

Image: ICMAT

From left to right and top to bottom, Patricia Fernández de Lis, who was the moderator, Rosina López-Alonso, María Blasco and Zulema Altamirano.

"the life-work balance and maternity are extremely important for women, and that's why we are also the ones who value stable employment and flexible working hours very highly. We are unable to devote so much time and effort to research work."

The high number of women who leave research is also due to the heavy administrative workload that usually falls on their shoulders (as well as on their male colleagues), to sexual discrimination, hostile work environments and sexist behaviour. "Little value is accorded to what women do, or they are condemned to uncreative activities or those involving little risk, not much beyond secretarial work," said the director of the Women and Science Unit of the Ministry of Science and Innovation, Zulema Altamirano, during her intervention at the meeting. The study on the situation of women in scientific careers in Spain produced by her department identified these and other reasons for why so many women abandon their careers, such as long daily working hours, the difficulty this causes in their role as carers, the salary gap, the difference between the importance given to the results obtained by them and their male colleagues and, given such difficulties, the social pressures involved in achieving promotion.

Understanding, awareness, training

"How come we still carry on talking about equality in science after so many years of different equality plans and women's commissions?," asked Altamirano. In her opinion, "we have to continue making greater efforts in order to understand the causes of this gender gap, while at the same time monitoring and auditing to check that the equality policies are working." She also believes that it is "important" to highlight not only women researchers themselves – their contributions and successes so they act as an inspiration- but also the fact that sexism really does exist in science.

Moreover, one of the key points is, in the words of this representative of the Ministry, to "embed concrete measures in the broader policy frameworks." A series of articles (one and another) were published in the journal *Nature* containing specific aspects that research centres should incorporate in order for women to develop their scientific careers fully and to enable them to reach leadership positions. Some of the most noteworthy of these proposals were flexible working hours, inclusive times for seminars, funded childcare, working online, and quotas earmarked for women in seminars and calls, among many others.

In order for environments in which science and research is conducted to be genuinely equitable, inclusive and diverse for the personnel engaged in these fields, "a structural change is necessary in every research centre," says a further participant in the meeting on equality, María Blasco, director of the Spanish National Cancer Research Centre (CNIO) and chairperson of the Alliance of Severo Ochoa and María de Maetzu Centres (SOMMa). In her opinion, "women should be in executive positions to enable them to help bring about this transformation towards a new framework, which would probably be better for men as well. The world of research has not been designed by us because we've never occupied decision-making posts until relatively recently."

"The CNIO is a feminist centre," said Blasco. The Women and Science Office was created in 2012 in order to make structural changes at the centre. At present, 70% of the research staff at the centre is composed of women, while it is men who are mainly at the head of the research groups. "We are increasing the number of contracts of women for leadership positions as a result of the demand for parity in the number of candidates who apply for such posts," said Blasco.

At the CNIO, a policy of quotas in the distinguished seminars and in their tribunals is being applied, 50% of which have to be conducted by women. However, this entails an "excessive workload for female researchers, especially in areas with a low presence of women, such as robotics," as Julia Borrás pointed out in a later debate. Borrás, who is a researcher at the Institute of Robotics and Industrial Computing and a member of the centre's Equality Commission (created in January 2021), thinks that the demand for parity among the candidates responding to calls, given its difficulty, is bound to meet with some opposition from many research groups. Faced with this challenge, María Blasco has no doubt what the solution should be: "Sometimes it's necessary to leave a call for applications null and void. It's the only way to achieve equality, because without equality there can be no excellence."

Image: ICMAT

The II SOMMa Equality event was organized online by ICMAT. In the image, from left to right and top to bottom, the opening table with Ana Bravo, José María Martell and María Blasco.

ICMAT and equality

"There are very few women engaged in mathematics." This is the bottom line as indicated by José María Martell, director of the ICMAT in his introduction to the 2nd Meeting on Gender Equality of the SOMMa centres and units of research excellence. He further stated that "our centre is firmly committed to the promotion of equality in this field by putting equality policies in place, making women in science more visible and encouraging scientific vocations among girls."

The Institute created its own Equality Commission in 2016, with the aim of "clearly identifying the causes of this imbalance and seeking optimal strategies to mitigate it," according to the chairperson of the Commission, Ana Bravo, who also spoke at the meeting. Also included under this umbrella are initiatives such as Science by women, a mobility scheme for female African researchers belonging to the Women for Africa Foundation. A further initiative, undertaken to raise visibility, is the section 'She does maths' in the ICMAT Newsletter, which presents profiles of women mathematicians.

Furthermore, with the aim of encouraging scientific vocations among girls, the centre participates in the International Day of Women and Girls in Science (February 11th) and the Women in Mathematics Day (May 12th), among many other outreach initiatives. There is also the #SteMatEsElla programme, aimed at a university audience and which consists of mentoring, coaching, webinars and raising the visibility of role models in order to promote the access to scientific and business careers for women undergraduates, master degree and PhD students of mathematics.

These and other undertakings of the ICMAT Equality Commission have been recognized in 2021 with the Certificate of Accreditation for Gender Equality from the Consejo Superior de Investigaciones Científicas (CSIC). "This recognition motivates us even more to continue making progress in an issue where there is still much to be done," concludes Martell.

Covid-19, a further hurdle

During the worst of moments of the SARS-CoV-2 pandemic, women researchers published and reviewed fewer papers than their male counterparts. This was the conclusion arrived at in an article published in *Nature*. It also contained pointers as to why the lockdown affected male researchers less than their female counterparts; it was the women who took on the most responsibility for caring and for domestic chores, and this made it difficult for them to balance work with family life.

In the particular case of Spain, the situation was largely the same. During the SOMMa meeting, Zulema Altamirano remarked that men with child dependents published more, while the people who submitted the fewest studies were mothers with scientific careers. Altamirano's department conducted an online survey, one of the results of which was that "in spite of everything, the women were able to go on submitting papers, but at the expense of their personal time and a great deal of stress."

This situation is completely at odds with the fact that "women have been vital in the effort to tackle the pandemic", remarked María Blasco in her address to the meeting. The reference is to scientists of the stature of Shi Zhengli, a Chinese virologist at the Wuhan Laboratory and an expert in coronaviruses coming from bats, who was among the first scientists to sequence the virus responsible for covid-19. And also to the Hungarian researcher at the University of Pennsylvania (USA), Katalin Kariko, who developed the technology on which the Pfizer and Moderna RNA vaccines are based.

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INTERVIEW: Eva Miranda, an invited speaker at the 8th European Congress of Mathematics

"I'd bet on Terence Tao resolving the Navier-Stokes conjecture"

Eva Miranda is a full professor at the Universidad Politécnica de Cataluña (UPC)-IMTech and a member of the Centre de Recerca Matemàtica (CRM). Winner of an ICREA Academia Prize in 2016, one year later she was distinguished with a *Chaire d'Excellence* from the Mathematical Sciences Foundation of Paris. She thereby became the first Spaniard and only the second woman in the world to occupy such a position. She has been an associate researcher at the Paris Observatory and is a collaborating researcher with the ICMAT. Some months ago, together with Robert Cardona (UPC), Daniel Peralta-Salas (ICMAT-CSIC) and Francisco Presas (ICMAT–CSIC), she published a paper in which for the first time the authors managed to construct solutions for a fluid capable of simulating any Turing machine. In June, Miranda spoke about these results as an invited speaker at the 8th European Congress of Mathematics (ECM), the meeting of the most important discipline on the continent and the second most important in the world. We talked to her about the reasons for her participation in this congress.

Ágata Timón García-Longoria

You, Xavier Cabré (UPC–ICREA) –as a plenary speaker– and Joaquim Serra (University of the Balearic Islands) –a winner of a European Mathematical Society Prize– were the only three Spaniards to be invited to speak at the congress. What was this experience like?

The invitation came as a surprise. I wasn't expecting it at all. The congress was scheduled to take place in 2020 but was eventually postponed until 2021, when it was held in a hybrid format with most talks conducted online. Firstly, congratulations to the organizers for going ahead with the congress in such difficult circumstances. Compared with how an ECM is usually held, on this occasion few people hooked up for the talks. And obviously the interaction with other colleagues was less intense than would have been the case at an in-presence congress. In a congress as big as this you normally have the chance to get to know people from other fields and other countries, to talk about subjects across the board, but this time we were unable to do that. We hope that the next ECM will be an in-presence affair. I'm sure that Seville, which is hosting the next congress, will organize an exceptional ECM.

How do you think the pandemic has affected mathematical research?

I think it's been a very difficult time for young researchers who are finishing their theses or are currently on a postdoc contract. They make up an invisible community and during this period have been even more invisible. It's also been complicated to start new collaborations or to work with collaborators who are not familiar with online tools. For myself, however, it's been a productive time, and fruitful in the sense that I've been able to carry on and explore more deeply some important subjects, as well as going ahead with supervising seven doctoral theses.

"Our result enables us to determine that certain hydrodynamic phenomena that are undecidable"

Could you tell us something about the result you presented at the ECM?

I spoke about our construction of solutions for fluid equations, which are capable of simulating any Turing machine. The flows we build codify a basic operating system that is able to execute any computational algorithm; that is, we could say we have invented a "water computer". One of the remarkable consequences of our result is that it enables us to determine that certain hydrodynamic phenomena are undecidable. In other words, there exists no algorithm to decide whether a fluid particle will pass through a certain region of space in finite time. We have shown that complexity of various types arises in fluids, from the dynamical, computational and logical points of view. This even occurs in some fluids that, a priori, may appear to be simpler because they correspond to stationary equations (i.e., not time-dependent).

How is this question related to the Navier-Stokes problem?

Terence Tao [professor at the University of California, Los Angeles, and a Fields Medal winner in 2006] has proposed a new way to attack the problem concerning the conjecture on the regularity of the Navier-Stokes equations, which is based on these fluid computers. He believes that it is possible to generate a blow-up in finite time by making use of constructions that are sufficiently complex, specifically, by simulating a Turing machine. This would imply that the Navier-Stokes equations do not always admit smooth or regular solutions, and that they are subject to sudden jumps.

One of the remarkable aspects of your work is the combination of different mathematical viewpoints: geometrical techniques, Turing machines, fluid mechanics, etc. What did the process of uniting these ideas involve?

We have been exploring a very interesting topic in which topology interacts with Turing machines. This is an explosive combination. For a long time I have been fascinated by the work done by Alberto Enciso and Daniel Peralta. Their research combining fluid dynamics and differential geometry techniques really drew my attention. In 2018 Daniel came to Barcelona to

Image: ICMAT

Eva Miranda works on different topics of differential geometry, mathematical physics and dynamic systems.

give a course on dynamical systems at a school and we began collaborating together. We extended his idea of switching from differential fields to differential forms in the area of b-forms, which are used to model manifolds without boundaries. This is something I had been working on intensely since 2010 opening a new brank in Poisson Geometry on b-symplectic manifolds. This correspondence –between Beltrami fields and Reeb fields- allows you to translate questions about fluids to geometry. It is like a mirror that reflects questions about fluid dynamics as questions about geometry. We managed to obtain an initial result by starting from a fluid in a manifold with a boundary, and next to the boundary we interpreted it as a b-contact structure. This is the precise moment when we began intermingling our two worlds.

Later, while travelling on a TGV I was reading Tao's proposal in his blog, and thought that maybe the ideas we had developed could be applied there. In the Beltrami- Reeb mirror, Tao's question about the universality of the Euler equations became a problem of geometry: Could any vector field be embedded as a Reeb field in a higher dimension? We could not resist the temptation to try to address the problem with our geometric methods, and we managed to do it.

How did the other members join the team?

In our geometric language, Tao was asking the following question: Can any vector field be a Reeb field (maybe in higher dimensions)? Of course, this was a question about flexible structures. We realized that we were going to need the so-called h-principle (h for homotopy), and we also recruited Francisco Presas, an expert in the subject, to the team. We were not thinking about Turing machines at that time, but since Tao had been using them and their relation with dynamical systems, we decided to look into them more closely. Then we realized how explicit that relation was; every motion of a point in the flow could be regarded as a Turing machine calculation. We started to visualize these Turing machines as a dynamical system. It is a very rich vision, because you can see those transformations as a time flow or a field flow. Robert Cardona, who completed his PhD last year under my supervision, has specialized in all this area closer to computer science. We had to go over many preliminary constructions before, after many attempts, we reached the construction proposed in the paper.

How did you arrive at the result?

First, we obtained a construction of universality. In the paper, which is still awaiting acceptance, we include a full Turing construction in a sphere of dimension 17. It seemed a beautiful construction to me, but after speaking about it at a seminar in Zurich in November, 2019, I realized that to capture the attention of analysts we ought to get a construction in three dimensions. However, the techniques we had used heretofore did not work in low dimensions, so in December, 2019, we set to work with different techniques. We had the construction in our heads, but we had to make the part corresponding to the Turing machines fit in with it, and that took quite a long time.

What has been the impact of this work?

I would say that it has been quite remarkable. On one hand, it is one of the papers that have been most downloaded from the Proceedings of the National Academy of Sciences (PNAS), where it was published, and has had quite an impact in the media. It first attracted attention in *El País*, *Pour la Science* and other media, and the result has also been mentioned in different leading scientific blogs, such as Gil Kalai's blog. Tao has recently given a couple of talks about it at the Mathematical Sciences Research Institute (MSRI) in Berkeley (USA), and at the BIRS centre in Banff (Canada), where he spent 15 minutes speaking about our result. He also mentioned our construction during the award ceremony of the Riemann Prize which took place last September.

What progress do you think Tao's proposal for solving the Navier-Stokes problem will make?

If I had to make a bet, I'd say that Tao is going to be able to solve the Navier-Stokes conjecture with a counter-example.... He's certainly been mulling it over with very innovative ideas, at least since 2017. I don't know to what extent he's going to use our machine, but obviously it's given him a lot of food for thought. In our result, the metric is one more variable of the problem, which provides a lot of freedom for making all these constructions. Tao says that by imposing certain restrictions on the metric and obtaining a condition of invariance, he believes it would be possible to arrive at a construction of this type. Since I believe in happy ends, I think he's going to do it. I think it would be very interesting for us to meet him in person, and quite likely lead to a collaboration.

You have still been moving forward with these ideas?

Yes. The conclusions we considered in the previous paper were about Beltrami fields; that is, a stationary case, which doesn't depend on time, and now we've managed to extend the Turing machine to the general Euler case (time dependent case). Everything has a price; now the dimension is increasing rapidly. What's more, we've used a result that Francisco Torres de Lizaur obtained just after, related to our paper in the PNAS. We happened to be working on that construction when Paco was finishing his article, and that was exactly the data we needed.

"We have managed to extend the Turing machine to the general Euler case"

What else are you working on at the moment?

One of the things I have been working on for some time is the extension the theory of symplectic topology to Poisson manifolds. The motivation for this can be explained by celestial mechanics, particularly with the restricted three-body problem. We study a systems consisting of a satellite, the Earth and the Moon. An attraction exists between them, but we assume that the satellite has negligible mass, so, in a way, what counts is the attraction between the Moon and the Earth. The motion of a satellite follows equations that we know, but if a collision occurs, or the satellite escapes and goes to infinity, the behavior of the satellite ceases to correspond to the Hamiltonian equations associated with the standard symplectic structure, and b-singularities appear in the symplectic structure. My interest lies in understanding these escape orbits. Now I am wondering when such orbits can appear. Can we prove that they always exist? This is interesting from the point of view of contact geometry, because it is related with the Weinstein conjecture on the existence of singular periodic orbits, which is a generalization of the standard Weinstein conjecture.

"My interest lies in understanding escape orbits that appears in problems of celestial mechanics"

What applications do these ideas have?

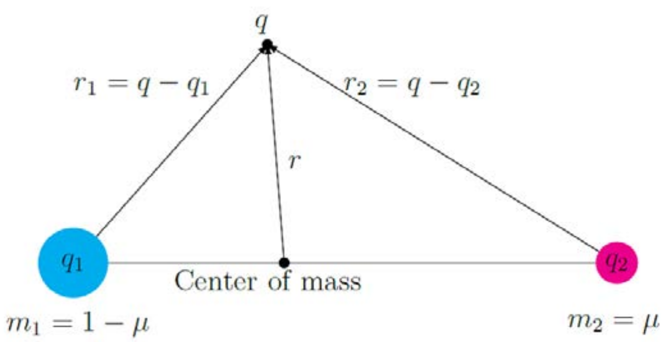
It's interesting to study these escape orbits because we don't want them to occur, but if they do, we want to be able to control them. So we apply the perturbation theory, also known as the KAM theory, to the problem of the motion of the satellite. There are many questions concerning three-body motion that are used in the practice of astrodynamics. For instance, the identification of the so-called Lagrange points, which are used to optimize fuel consumption; if you navigate close to these points you use less fuel.

What results have you obtained from this?

With Daniel Peralta, and with Cédric Oms, a former student of mine, we demonstrate that by using the Beltrami-Reeb mirror there exist a minimum number of orbits, dictated by the form of the set of the line to infinity. The topology of the critical set determines the minimum number of orbits that may appear. This result can't be applied to the three-body problem because there are certain initial hypotheses that are not fulfilled, but we're trying to improve it. Demonstrating it in general is one of the questions I'm working on right now. What's more, with Cédric I obtained a new family of periodic orbits for the three-body problem that was unknown until now. I am very proud of this.

This work has in some way converged towards the work on fluids.

Yes, they were two topics that apparently were unrelated, but it turns out that they are and the nexuses are diverse. On the one hand, any problem involving Beltrami field orbits can be seen as a problem of orbits for Reeb fields by means of the Beltrami-Reeb mirror. In particular, the results known as the Weinstein conjecture (whenever known to hold) can be applied to prove the existence of periodic orbits for Beltrami fields. The mirror works in both directions. So we've been able to apply a result by Karen Uhlenbeck (Abel Prize winner) on the genericity of the Laplacian eigenfunctions in order to demonstrate the existence of orbits with singular points on the other side of the mirror. In particular, the escape orbits present this behavior. What's curious is that the escape orbits can be seen as singular orbits (sometimes periodic). This brings us back to the terrain of the Weinstein conjecture.



The restricted three-body problem is one of the topics on which Miranda has made contributions in recent years.

You're also working on Floer problems.

Yes, I was interested in questions about periodic orbits, which is a question about dynamical systems, and that led me to the Floer problems, which concern symplectic and contact topology. Andreas Floer developed an algebraic complex for studying the existence of periodic orbits of Hamiltonian fields. He was looking for a characterization of the existence of orbits, and that was the driving force behind the Floer theory. He believed that this theory was going to depend on the Hamiltonian, but he ended up by demonstrating that wasn't the case. Even though, at first, that must have been very frustrating, it was precisely what eventually enabled him to prove the Arnold conjecture, which deals with the fixed points of a diffeomorphism. Now I'm trying to translate what Floer was doing in order to identify certain types of periodic orbits in which are marked points at which the field vanishes. The motivation comes from celestial mechanics, precisely the problem I explained before, because some of these singular orbits correspond to escape orbits.

Apart from that, you're also interested in quantization.

Yes, that's a more classical problem. We are trying to combine geometrical models with models of quantum physics. There are highly classical open conjectures that pose the question that if, when you have symmetry in a certain system, quantization commutes with reduction. We are extending the models to more general situations, particularly to Poisson manifolds, which respond to the existence of equilibrium points in physical problems or to parametric versions of classical systems, where the parameter may be singular. Obtaining models that are compatible with physics is complicated, but not impossible.

In addition to your research work you also supervise doctoral theses. How important is this task?

I'm a happy thesis advisor. Six of my students have already defended their PhD theses and three more will do so soon. Collaboration with my students is often very long-lasting and I learn a lot from them. I feel rewarded when I see them progress in their careers and acquire more experience. Last year I had seven PhD students simultaneously, which perhaps was a bit too much. Four of them defended their theses during 2020-2021, and there are three more still to fly the nest, plus another who is about to start.

When did you realize that you wanted to devote yourself to mathematics?

I liked literature very much as well as mathematics; they have something in common, and that's creativity. For me they were a way of escaping from reality. Mathematics provided me with an inner safe haven, something that gave me a sense of freedom. I had a really good time writing as well as solving mathematical problems. When the time came to make a decision, I opted for mathematics, although I often wonder what would have happened if I had chosen the other path.

How did you get interested in research?

In my third year, our differential geometry professor set us a problem in foliation theory, which studies how it is possible to slice a space into pieces in a differentiable way. It fascinated me because it seemed something beautiful and artistic, and I told him I wanted to know more about the subject. He lent me a book on foliations that I found very exciting (*Geometric theory of foliations* by Camacho and Lins Neto), and I was able to study some related subjects and tackle some more advanced questions during my career. When I completed my degree I chose this differential geometry professor, Carlos Currás-Bosch, as the director of my PhD, so finally I was able to realize my dream and do a thesis on lagrangian foliations, the objects that had fascinated me in my third year.

What did you do after finishing your thesis?

I went to France as a postdoctoral researcher and stayed there for three years, nearly two of them on a Marie Curie contract, and then I came back with a Juan de la Cierva contract. Coming back was very hard and full of uncertainty. The reintegration path for postdocs is something that remains unsolved in Spain. We are asking your researchers to go abroad, but the professional opportunities open to them when they want to come back are just not competitive enough. However, the Margarita Salas and María Zambrano calls for applications have been a breath of fresh air. These calls ought to be made on a regular basis in order to ensure that future generations of researchers are retained long-term in Spain. We have to look for solutions to this issue so that the people who we have trained and educated here are not lost. Our university staff is getting very old and it's vital to let in new talent, which will contribute fresh energy and new ideas to the system.

In 2016 you were awarded an ICREA Academia prize, and in 2017 a chair of excellence from the Mathematical Sciences Foundation of Paris. What impact have these distinctions had on your research career?

I can say without any doubt that both awards have helped to improve the quality of my research work. On the one hand, the ICREA Academia prize completely changed my life. For five years, I've been able to devote myself intensively to that research topic and tackle challenges that would have been impossible in any other way. In particular, I find it hard to believe that we could have managed to construct the Turing machine without all that time dedicated to research. And I don't think I could have found the new family of periodic orbits of the restricted three-body problem either, or demonstrate the desingularization of b-manifolds theory, not to mention a long list of others. What's more, the financial resources provided by ICREA Academia has enabled me to increase my research team with the incorporation of PhD students, young scientists just starting out on their research careers, and even a newly created postdoc position in the middle of the pandemic, when few such posts were available.

Furthermore, the Mathematical Sciences Foundation of Paris chair of excellence was undoubtedly a distinction of the first rank that allowed me to work side by side with the leading exponents in the study of celestial mechanics problems at the Paris Observatory, which opened up new horizons in the study of b-manifolds and gave me much inspiration. I was also able to bring two of my PhD students to Paris for a semester as well as funding a closing colloquium of the chair to which many collaborators were invited. I have also maintained a close relation with the Paris Observatory and remained affiliated as a researcher.

Image: Eva Miranda

An artistic representation of the singular orbits studied by Miranda appears in the reflection of the Rakotzbürke Bridge in Germany. The singular points in the semicircle are the intersection with the water.

You were the first woman mathematician to be distinguished with an ICREA Academia award and only the second to attain a Mathematical Sciences Foundation of Paris chair of excellence.

That's right, although since then Teresa Martínez Seara and Núria Fagella have also received an ICREA Academia prize. I'm pleased not be the only woman in this category. As regards the FSMP chair of excellence, this is very surprising, and only Hélène Esnault and I have received one. Hélene received it in 2011.

How would you describe the current situation of women in mathematics?

Little has been done to improve the situation of women in research. We're aware that the problem exists, and we organize sessions devoted to addressing of the issue, as well as round-table discussions and talks, but so far no solutions to remedy it are being put into practice. It's not enough to just talk about it. We ought to be able to gain more ground and get more women to take up careers in research, otherwise we're wasting a great deal of talent. Diversity in science is vital and talented women should be included to achieve advances, and to do that it's necessary to make some bold decisions.

What might be some possible solutions?

I think that if we raise the profile of women we'll make a lot more progress. For example, by proposing more women as plenary speakers at congresses or more women as candidates for prizes, we'd make them far more visible. Unconsciously, it's always men who are considered for prizes and we have to go much further than that, changing this mentality and promoting women more to the forefront in science. Right now the position of women in science is still under question. We sometimes have to make twice the effort to get the same recognition as our male colleagues. If a woman's name comes up for consideration for a prize or as a plenary speaker, voices are often raised against it. We have to stop questioning women in science and put them forward for talks and prizes, without backing down and without fear of making a mistake. Everything would be easier for all of us if this was normalized.

ICMAT QUESTIONNAIRE: Tony Carbery (University of Edinburgh)

Tony Carbery, a full professor at the School of Mathematics of the University of Edinburgh, is one of the new researchers who is taking part in the ICMAT Severo Ochoa Distinguished Professors programme 2020-2023. For that reason, Carbery has been working at the centre since October 2021, getting on with research and collaborating with Javier Parcet (ICMAT-CSIC) and José María Martell (ICMAT-CSIC)'s group.

Laura Moreno Iraola

Why did you choose mathematics ahead of any other subject?

At school I liked Physics, Chemistry and languages including French and Spanish. I was good at Maths, and I found its structures appealing, but I didn't see myself ending up in it. But the more science I studied, the more I felt that it wasn't *getting at the truth*, and indeed on occasions I realised that certain obfuscations were being made, deliberately or otherwise. So for the last two years of high school I aimed to study mathematics, Spanish and French, but the timetable didn't allow it. In the end I went for Pure Mathematics, Applied Mathematics and Physics. From that point on I had little flexibility left! So I studied Mathematics at University of Oxford, but I've managed to maintain my interest in and love for the Spanish language.

How was your first encounter with mathematical research?

In the old days there were no undergraduate research projects, so my first encounter with research was necessarily as a PhD student under my supervisor John Garnett at UCLA. Just before Christmas in 1980, John presented me with a pile of large survey papers on various potential research topics to read over the vacation. One of these papers was Antonio Córdoba's "Translation Invariant Operators" and I fell in love with it. After Christmas I told John I wanted to work in that area. He replied: "That's a pity, I don't know anything about that area." Nevertheless, we pressed on, and a few months later John was at a conference in Chicago at which Antonio presented a list of open problems (this was long before PhD students were encouraged to attend conferences themselves.) Luckily I managed to solve a variant of one of the problems Antonio had mentioned, and things began.

What did you like most about your early experiences with mathematical research?

It was a mixture of wonderment, excitement and frustration. There were so many things I wanted to understand, and so few that I could. Still, you can make a career by understanding a few decent things a year, and I slowly realised this. For me, the human element was really important. First, the tremendous support given to me by John Garnett, which I've tried to reciprocate with my own PhD students. Then, the interactions with other young researchers in Chicago, especially Fernando Soria, with whom I developed a lasting collaboration and friendship which led me to visit Spain almost all the time from the mid 1980's onwards. And of course the conferences, especially the legendary 1983 El Escorial meeting which, for me, has to be the best of my whole career.

Which scientist impressed you most during your career?

Limiting myself to mathematics, and then to the mathematics that I know, it has to be Eli Stein. At John Garnett's behest, I read Eli's classic *Singular integrals and differentiability properties of functions* as a PhD student (John declared it was his favourite book) and was immediately captivated by it and its contents. Meeting Eli at the 1983 El Escorial conference, and finding that this God of Mathematics was also an approachable, warm and interested human being further reinforced for me the importance of human interactions within mathematics. And this is before we begin to speak about the huge range of things which piqued Eli's interest. Above all, Eli encouraged cooperation rather than competition in research. Although Calderón and Zygmund were undoubtedly the founding fathers of modern harmonic analysis, the enormous scope, influence and importance of the area, and its state of robust health over the last 60 years, is in no small measure due to Eli and his superb mathematical taste. We all owe him a tremendous debt.

Do you have a particular theorem or formula you especially like?

I tend to think more about the things I don't understand than those that I do. So I have lots of favourite problems. The theorems I like best are those which pose new problems. So, for example, Carleson's almost-everywhere convergence theorem for square integrable functions on the line (what happens in higher dimensions?); Guth's endpoint multilinear Kakeya theorem (what about restriction?); and the results of Komlós, Pintz and Szemerédi on the Heilbronn triangle problem (which are not definitive and demand further attention).

What is your favourite mathematical book?

Despite it is *vintage, Singular integrals and differentiability properties of functions* by Eli Stein.

How would you describe/sketch your research interests in a few lines?

I'm still motivated by the same broad area of research as ever: the interplay between the operators of harmonic analysis and geometry, whether that be in the form of curvature or discrete geometry/combinatorics, and also the relation with functional analysis.

Which recent results in your field would you highlight?

Although it's not quite harmonic analysis per se, I find the recent work of Tidor Yu and Zhao on joints and multijoints for affine planes (and indeed algebraic varieties) of arbitrary dimensions to be very impressive. Of course, it builds on a long line of earlier work, notably that of Ruixiang Zhang.

Which particular mathematical problem do you consider especially challenging?

For me, it has always been the same one: Stein's restriction problem for the Fourier transform. It has fuelled so much really high-quality mathematics, and once again demonstrates Stein's incredible insights into the field and supreme good judgement and taste, but yet still after more than 50 years it remains unresolved. One thing I do know: it won't be me who solves it!

You are one of the ICMAT Distinguished Professors, what is your work regarding this position about?

I hope to contribute both to the day-to-day life of ICMAT while I'm in residence, via seminars, working groups and informal discussions, and also to its visibility via helping to organise some high-profile in person visits and research events once the pandemic permits. And, of course, to collaborate with the researchers at ICMAT.

For that reason, you are currently visiting the ICMAT, what would you highlight most of the Institute and the environment?

So far, ICMAT has offered a wonderfully peaceful environment to get on with my research. As the pandemic (hopefully) recedes, I hope it will offer even more in the way of human interaction and potential for collaboration.

Which subjects in mathematics outside your field would you like to learn more about?

Given its current influence on harmonic analysis, it would have to be algebraic geometry (but it's not easy to teach old dogs new tricks...).

Besides mathematics, which activities do you like most?

Swimming, walking, travelling, the Spanish language and culture, rock music, classical music, food, wine, beer and conversation with friends, setting the world to rights...

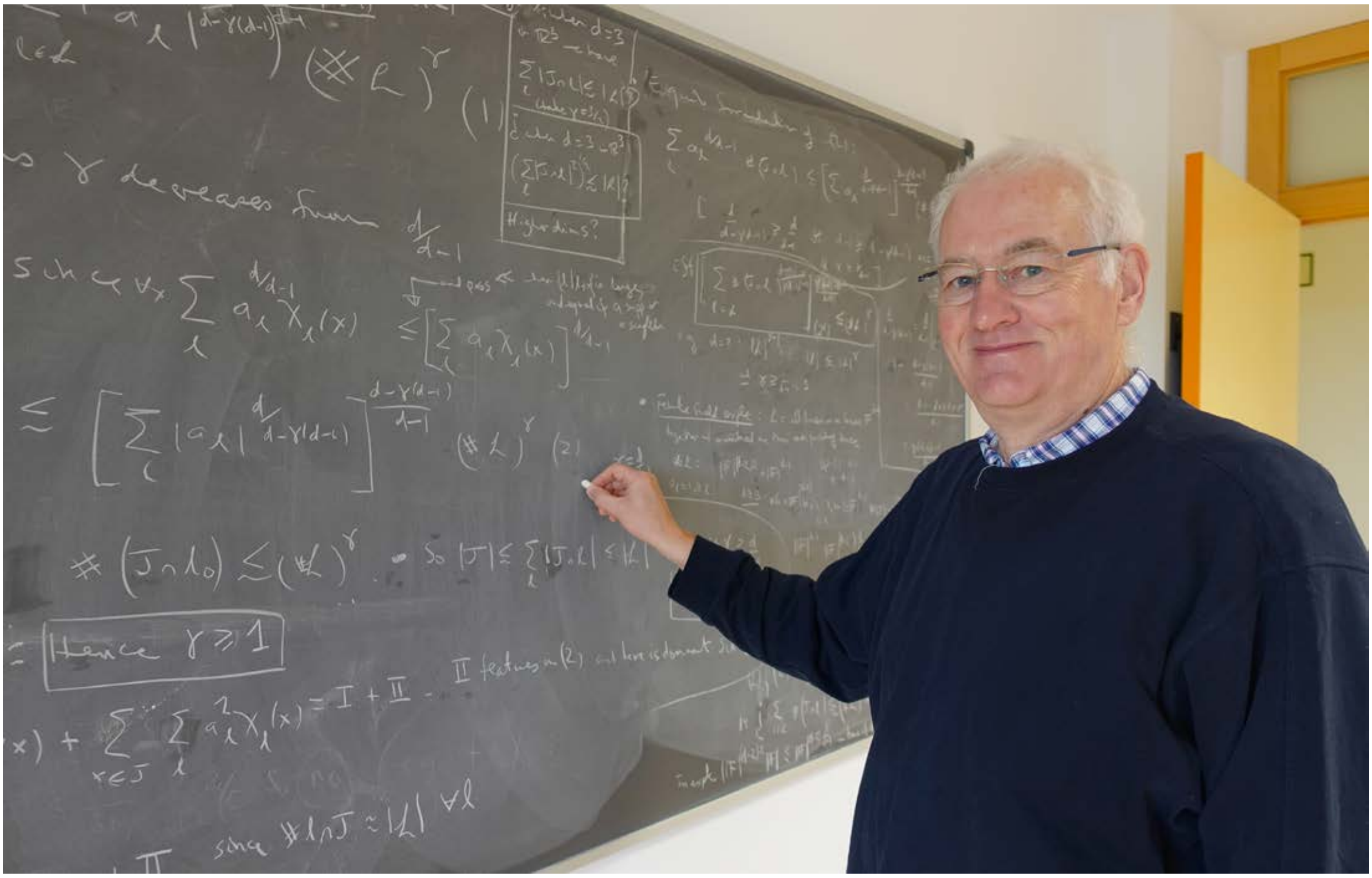


Image: ICMAT

Anthony Carbery, a professor at the School of Mathematics of The University of Edinburgh, is a leading expert in harmonic analysis.

TELL ME ABOUT YOUR THESIS: Francisco Mengual

Title: "Instabilities in fluid mechanics and convex integration"

Author: Francisco Mengual

Supervisors: Ángel Castro and Daniel Faraco

Date: July 16th, 2021

Francisco Mengual's thesis focuses on the study of various problems in fluid mechanics related to turbulence. The aim is to describe the dynamics of fluids on the basis of a hydrodynamic instability concentrated on an interface. To that end, a *turbulence zone* is introduced that grows around an interface. The construction of these turbulent fluids is based on a new version of a convex integration method proposed in hydrodynamics by Camillo De Lellis and László Székelyhidi.

Francisco Mengual

Turbulence is a physical phenomenon with which we are not unacquainted. Most of us have experienced it at some time in our lives, whether it is on a plane when we are asked to fasten our seat belts or at sea when a wave buffets us towards the shore. It can also be observed every morning before leaving the house; one only has to pour milk into the coffee and see how the two liquids begin to mix and form very curious patterns. In fact, one intrinsic characteristic of turbulence is that these patterns can be observed in a broad range of scales, from small, as in our cup of coffee with milk, to the enormous, as in solar flares (which cause the northern lights) or the explosion of supernovas.

These examples give us an idea of what turbulence is like; sudden capricious changes in the physical properties of a fluid (such as density and velocity). Otherwise, we say that the fluid behaves in a *smooth* way. Mathematically, fluids are modelled by means of partial differential equations. The development of mathematical techniques enables some qualities of these equations to be deciphered, such as the existence, uniqueness and properties of their solutions.

The case of turbulent fluids continues to be a mystery, which makes it impossible to arrive at long-term predictions about their evolution. Despite the great efforts made by scientists, from the sketches drawn by Leonardo da Vinci (500 years ago) to the most recent theories by Osborne Reynolds, Andréi Kolmogórov and Lars Onsager (to name just a few), at the present time no complete theory exists for describing turbulence in a satisfactory way. According to a winner of the Nobel Prize in Physics, Richard Feynman, this remains the most important unsolved problem in classical physics.

In my thesis, we have studied various problems concerning turbulence. These problems involve a fluid (in two dimensions) that is smooth in two regions of the plane and separated by a curve, which we refer to as the interface, at which a discontinuity occurs. For example, we may consider two different fluids (coffee and milk) and the interface that initially separates them. In some cases (for instance, if the heavier fluid is above the lighter one), this interface is unstable. This means that small perturbations at the interface, imperceptible to the naked eye, end up increasing considerably, thereby forming as they flow patterns such as those mentioned above. This suggests that it may be possible to describe the dynamics of the fluid as the evolution in three regions; two zones where the fluid continues to be smooth and one "turbulence zone" that begins to grow around the interface, where the fluid behaves in an irregular way.

While the mathematical description of these turbulent fluids seems to be practically impossible due to their unpredictable nature, it can be achieved by adding highly oscillating terms (within the turbulence zone) to a more regular fluid, with the aim of describing the macroscopic properties. The solutions to these modifications of the equation are given the name of subsolutions. This construction is based on the innovative version of the convex integration method introduced into hydrodynamics by Camillo De Lellis and László Székelyhidi in 2009. This technique, the origin of which dates back to the work of John Nash (1954) and Mijaíl Gromov (near 1970) in differential geometry, has proved to be very robust and flexible. It is important to point out that it has enabled some problems that are highly relevant to turbulence to be solved, such as the conjecture by Lars Onsager (Philip Isett 2018) and the non-uniqueness of solutions to the Navier-Stokes equation (Tristan Buckmaster and Vlad Vicol, 2019). Furthermore, it has been possible to apply the technique to a multitude of models in fluid mechanics: the transport equation; some active scalar equations (such as IPM and SQG), the Euler equation (compressible and incompressible flow); the Navier-Stokes equation, and the MHD (Magnetohydrodynamics) equation, among others.

The solutions obtained with the convex integration method coincide with the subsolution outside the turbulence zone and, in particular, are smooth. However, within the turbulence zone there are infinitely many solutions and their behaviour is irregular. Despite the lack of uniqueness and regularity within the turbulence zone, together with Ángel Castro and Daniel Faraco, we prove that by refining the convex integration method it is possible to select those solutions that are practically indistinguishable from the subsolution at a microscopic level. This recovers in some respects the uniqueness of the dynamic and emphasizes the role of the subsolution as the macroscopic part of the fluid.

A further key aspect for studying turbulence is the vorticity of the fluid. The presence of vorticity and turbulence are related, specifically when the vorticity accumulates in small regions of the space. A typical example appears when two atmospheric bands advancing at different velocities come into contact. This discontinuity induces vorticity at the interface separating them, which triggers a turbulence zone where one may observe a pattern of spirals that roll up. This phenomenon is known as the "Kelvin-Helmholtz instability". The incompressible Euler equation is used to model this phenomenon, starting from a vorticity concentrated on a curve (like a Dirac delta), which is known as the "vortex sheet". The dynamics of the vortex sheet are used to model some physical phenomena observed in the atmosphere and the oceans, such as the aircraft wake and oceanic mixed layer. This is a classical problem of fluid mechanics and has been studied in two different ways. The first consists in analyzing the evolution of the curve where vorticity accumulates. However, with this approach the problem is poorly posed –in general, the vorticity cannot continue to be accumulated on only one curve– and can only be resolved if the initial datum is analytic. The second approach consists in studying the evolution of the vorticity. In the case of vorticity with fixed sign, Jean-Marc Delort constructed global solutions (in fact, for a class of more general vortices) in 1991.

The case of vorticity with mixed sign was an open problem that was also interesting in itself, both for its practical applications in aerodynamics and for the complex structures in the intermingling of regions with positive and negative vorticity. By means of the convex integration method, together with Lászlo Székelyhidi, we construct solutions to the vortex sheet problem, the vorticity of which is non-analytic and whose sign may change. Furthermore, this approach enables the relation between the dynamics of the turbulence zone and the dissipation of kinetic energy to be established. It is important to point out that this anomalous dissipation of energy is a prediction of the so-called zeroth law of turbulence, which states that in turbulent regions an anomalous dissipation of energy is produced that is independent of the viscosity when this tends to zero.

Another classical problem we addressed in the thesis was to determine the evolution of two incompressible fluids (for example, water and oil) in a porous medium. This is known as the Muskat problem, the origin of which dates back to studies on petroleum engineering. When the heavier fluid is completely beneath the lighter, the interface is stable and its dynamic is well defined. However, when the heavier fluid is completely above, the interface is unstable; small perturbations at an analytic interface end up by creating a finger pattern where the fluids appear to mix. This phenomenon is known as the "Rayleigh-Taylor instability". The original motivation for the thesis was to study the partially unstable Muskat problem.

In 2016, Castro, Córdoba and Faraco constructed solutions for the totally unstable Muskat problem by introducing a dynamic *mixing zone*. To that end, they combined for the first time the dynamics of the interfaces with the convex integration method by means of a veritable work of goldsmithery. In 2012-2013, Ángel Castro, Diego Córdoba, Charles Fefferman, Francisco Gancedo and María López-Fernández proved the existence of an analytic interface in the totally stable regime, which evolves until it becomes a partially unstable analytic interface (in which the heavier fluid is above in a region of the interface), and later the analyticity is lost at a point of the unstable region. Together with Ángel Castro and Daniel Faraco, we demonstrate the continuation of the dynamic after this singularity. In this case, the implementation of the convex integration method requires the localization of the mixing zone around the unstable region and to show its compatibility with the parabolic analysis in the stable region.

Finally, we also study the Muskat problem on the basis of two fluids with different densities and viscosities. In this case, we check that the connection, as established by László Székelyhidi for the case of constant viscosity, between the subsolution and Felix Otto's Lagrangian relaxation solution, is also satisfied in the case of the viscosity jump. Additionally, we show how a singularity in the relaxation prevents the fluids from mixing wherever there is neither Rayleigh-Taylor instability nor vorticity at the interface.

All this work provides further proof of the efficacy of the convex integration method as a tool for deciphering hidden structures in the physical equations. Moreover, this understanding helps us to explain physical phenomena of great practical importance. Many open questions still remain, the answers to which will require the refinement of this method and the development of new mathematical techniques involving great complexity.

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TELL ME ABOUT YOUR THESIS: Patricia Contreras Tejada

Title: "Resource characterization of quantum entanglement and nonlocality in multipartite settings"

Author: Patricia Contreras Tejada

Supervisors: Carlos Palazuelos and Julio I. de Vicente

Date of defence: July 14th, 2021

Patricia Contreras Tejada's thesis focuses on a novel research subject: quantum systems of more than two particles. Specifically, she studies configurations of this type which are entangled –where a description of each particle is not enough to know the whole system– or nonlocal –where, on observing the particles, it is impossible to describe their correlations by means of classical models. In order to achieve particular effects in systems of more than two particles, networks of quantum particles are used, and these networks are also addressed in her thesis. Finally, Contreras Tejada has developed a physical principle that provides a constraint for ruling out post-quantum theories, which unites quantum theory with the scientific study of knowledge.

Patricia Contreras Tejada

Quantum theory is one of the most important scientific theories of the 20th century. Its mathematical characterization is experimentally well backed up and provides unprecedented knowledge about *Nature*, because it is the best explanatory and predictive theory developed to date. Some of the predictions of this theory appear to be completely counter-intuitive, since in many cases they arise from two phenomena that are exclusively quantum: entanglement and nonlocality. The study of these phenomena in systems of more than two particles, known as multipartite systems, is the main focus of my thesis.

Multipartite quantum systems have only recently come to the fore in the development of quantum theory. They are useful to understand the most fundamental components of *Nature*, to implement the huge amount of applications based on these types of systems as well as to develop new systems. Multipartite systems are difficult to control in the laboratory, so it is not easy to study them experimentally. That is why it is vital to subject them to a full theoretical analysis. However, given the mathematical complexity of their description, there has been less progress in this area than in that of two-particle, or bipartite, systems.

In the thesis we aim to determine which configurations of these systems are the most *useful*. While the notion of usefulness varies according to context, in general terms we aim to achieve systems with more entanglement or more nonlocality.

Entanglement is a property of quantum states, which are the mathematical expressions we use to describe these systems. A two-particle state is entangled if the description of each particle is not enough to describe the entire system and, instead, a global description is required. By measuring each of the particles that make up the state, it is possible to obtain the correlations between the results of these measurements. Nonlocality occurs when it is impossible to find a classical model (i.e., one that is not quantum) that explains these correlations.

The concepts of entanglement and nonlocality become more complex for quantum states consisting of more than two particles. We focus on cases of *genuine multipartite* entanglement and nonlocality which, for the purpose of this thesis, are the most useful properties we can find in a quantum system.

Among pure (i.e., noiseless) bipartite states, the most useful state is called the maximally entangled state. Ordering the states of more than two particles in terms of their usefulness gives rise to a hierarchy. This setting is known as *resource theory*.

Attempts have recently been made to extend this theory to multipartite states, but serious problems have arisen that make it impossible to arrive at a resource theory for these states using the transformations between quantum states that it is physically reasonable to allow.

Given this impediment, in this thesis we propose a new resource theory by extending the set of transformations allowed. Not only does it not lead to the aforementioned problems, but it also gives rise to a genuine multipartite maximally entangled state which is, moreover, unique. This constitutes a milestone that has not been reached before and which opens up new avenues for quantifying this type of entanglement.

Furthermore, in the thesis we also study networks of quantum particles that arouse great interest in the field of quantum technologies, because they constitute an accessible way of achieving multipartite effects without any greater technological requirement than that needed to entangle pairs of particles. Entangled states between pairs of agents are shared in these networks, in which each agent is connected to either one or more of the others.

We first assume that all the shared states are pure; that is to say, noiseless. In this case, it is known that every connected network of pure bipartite entangled states is genuine multipartite entangled (a necessary condition for being nonlocal). However: Which of these states give rise to genuine multipartite nonlocality? Surprisingly, this occurs for all networks; any connected network of pure bipartite entangled states is genuine multipartite nonlocal. This result reveals a fundamental property of such networks that was hitherto unknown.

Nevertheless, the noiseless case is not enough. It is vital to take noise into account when dealing with applications to quantum networks such as the quantum internet. Given this situation, we prove that not all networks are valid for these applications; unlike in pure-state networks, geometry plays a key role. The least connected networks cease to be entangled as the number of nodes for any level of noise increases. At the other extreme, the fully connected networks retain entanglement for any level of noise beyond a certain threshold and for any number of nodes. At a time when research into the quantum internet is on the rise, knowledge of its basic limitations is crucial to avoid pursuing the wrong track.

The last chapter of the thesis addresses a physical principle. It acts as a constraint for ruling out post-quantum theories that describe the experimental results available. This line of research may end up establishing that quantum theory is the theory which best describes the world as we know it. Physical principles enable us to integrate our knowledge of the natural world and to understand why the quantum world behaves as it does. The principle addressed in the thesis is inspired by a result that is highly influential in epistemics (i.e., the scientific study of knowledge). Specifically, we prove that two quantum observers cannot have *common certainty of disagreement* about perfectly correlated events. Within the classical framework, this impossibility is an indicator of epistemic consistency, by which we argue that the principle of non-disagreement should also occur in any theory of *Nature*.

Our contribution not only adds a physical principle to the list, but does so by uniting two fields of knowledge that have until now been regarded as unlinked: quantum theory and epistemics. This discipline has exerted a great influence on economics by characterizing situations in which various agents make bets on the basis of their own individual interests, each one of whom are in possession of different information. The research conducted in this thesis enables these results to be extended to a scenario in which the agents share quantum resources; for example, if they are connected to the quantum internet. These economic situations will be realistic in the near future.

The task of restricting possible theories of *Nature* by means of external principles is complex. We propose a step in this direction that also leads to close connections between quantum information and epistemics. The question naturally arises of confirming whether this principle is intrinsic to *Nature* or whether it is possible to generate experimental correlations that do not fulfill it. Furthermore, if list of physical principles that may characterize quantum theory were completed, this would endow the current best description of *Nature* with a very solid operational grounding.

Other questions arising from the thesis are: the possibility of measuring multipartite entanglement in a unique way; whether it is possible to generate genuine multipartite nonlocality using a single noiseless entangled state, and the geometry required to obtain genuine multipartite nonlocality with networks of noisy states.

SHE DOES MATH: Tatiana Toro (University of Washington)

Image: MSRI

Tatiana Toro has been named director of the Mathematical Sciences Research Institute (MSRI) for the period 2022-2027.

Tatiana Toro is a professor at University of Washington since 1996, and, recently, has been chosen as the new director of the Mathematical Sciences Research Institute (MSRI), in USA; she will take office in August 2022 for five years. Toro is one of the main experts in geometric measure theory, partial differential equations, harmonic analysis and calculus of variations fields. Thanks to her contributions and work, she could be one of the invited speakers in the International Congress of Mathematicians (ICM) held in Hyderabad (India) in 2010. Also, she was awarded with a Guggenheim grant in 2015, she was elected fellow of the American Mathematical Society (AMS) and as a member of the American Academy of Arts and Sciences in 2020, among others.

Research fields: Geometric measure theory, partial differential equations, harmonic analysis, calculus of variations.

"I had no idea what a degree in Mathematics involved, but I didn't really care," confesses Tatiana Toro (Bogotá, Colombia, 1964), a researcher at the University of Washington since 1996 and the recently elected director of the MSRI (Mathematical Sciences Research Institute, USA).

Toro realized that she liked mathematics at the age of six or seven, since she loved the manipulative games she was given to do at school, but she never thought she would end up devoting herself to that subject. Until she did her baccalaureate, she did not know that it was possible to study mathematics at a higher level. She remembers exactly the year, 1981, when she made the decision to do so, because it was then when she took part in the International Mathematical Olympiad as a representative of her country, Colombia. "I met a lot of Mathematics students there, and I thought if that option existed, I could do it too." It was the first time that a team from Colombia had taken part in the competition, and consequently Toro was the first woman from her country –and the only one until many years– to participate in the Olympiad. "Of all the 186 participants," she says, "I think there were only six women, and three of us went on to become research mathematicians."

Although she studied in Bogotá, Toro has spent the rest of her professional career in the United States. As she explains, "when I finished my degree, I knew that the only way I could go on learning and discovering more about mathematics was to do a PhD. Right away I met someone who I thought would be a good advisor, Leon Simon. I liked the subject and started it without much further thought."

The subject of her thesis, which she read in 1992 at Stanford University, concerned the field of geometric measure theory, an area she still works in today. She is also interested in partial differential equations, harmonic analysis and calculus of variations. "One of the things I've managed to do with some success is, together with my collaborators, transfer ideas from geometric measure theory to other areas of mathematics," she says. Her work consists of a combination of different approaches, of which she gives an example of a classical problem in geometric measure theory. "Let's imagine we have a wire in space and the wire is twisted. We want to find the surface with the smallest area whose boundary is this wire. The calculus of variations is concerned with establishing what the minimum of this area is and how we can find it. In order to do that, we need a class of objects for measuring whether or not the surfaces have a minimum area. This class is defined by means of the geometric measure theory, and to see if the objects are smooth or not we use partial differential equations."

Some of the projects she is currently engaged in are related to the following type of question: Given a number of disperse points in $\mathbb{R}^3$, is it possible to find a surface that passes through all these points and has good properties? Together with her collaborators, Toro has managed to find conditions that ensure that this is possible. One of the open questions in the field is whether the surfaces studied by Semmes admit bi-lipschitz parameterizations. Toro does not think that they do. "We're working on the construction of a counterexample of this," she says.

She has obtained a recent noteworthy result in collaboration with José María Martell, the director of ICMAT, Steve Hoffmann (University of Missouri), Svitlana Mayboroda (University of Minnesota) and Zihui Zhao (University of Chicago). This consists in a classification of the smoothness of a set in $\mathbb{R}^3$, in terms of the behaviour of the partial differential equations in this domain. "It is a result in which the equivalence between the geometric properties of a domain and the analytic properties of the solutions to partial differential equations in certain classes is established," explains Toro. "The process was very enjoyable. We happened to meet at the MSRI, and I remember very well the discussion we had in the open air. You can learn a lot from this kind of difference of opinions, or when your collaborators don't understand something and you have to make the effort to clarify whether or not it's really as you believed it was."

Tatiana Toro is currently doing research on open questions regarding geometries that are different from the euclidean norm; for instance, one in which the balls, rather than spheres, are like the balls used in American football. "There are very simple questions in this geometry that we don't know how to answer," she says.

One of the most notable moments in her career was her participation as an invited speaker at the 2010 International Congress of Mathematicians (ICM), held in Hyderabad, India. "Never for a moment did I expect to be invited. It had been a difficult year for me on a personal level and for that reason it meant such a lot more," says Toro.

In August 2022 she is due to take up her new post as director of the Mathematical Sciences Research Institute (MSRI), an institution in which she has been involved for years as a member of the scientific committee. "I believe that much can be done through the institute," she says, "such as continuing to stimulate scientific excellence." Not only that, but also to contribute to the development of mathematics and the mathematical community from different perspectives, such as education, outreach for the importance of mathematics or to attract groups under-represented in the discipline. "We want to pursue certain successful lines of action and extend them to the country as a whole, such as the summer programme with assistance for women mathematicians to enable them to devote themselves exclusively to research for two or three weeks, and the welcome programme at the institute for undergraduate students from communities that are traditionally under-represented in mathematical sciences", explains Toro. "In particular in terms of gender representation we're not where we want to be, or where we ought to be. Also that the pandemic might have had a negative effect on young women still in temporary positions. In one of the MSRI's committees, we're looking into what's happening with them," she concludes.

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PROFILE: Alba Dolores García Ruiz (ICMAT-CSIC)

"I'm learning to apply mathematics to real-world problems"

Image: Alba García Ruiz

Alba Dolores García Ruiz began her doctorate at ICMAT in October 2021.

Alba Dolores García Ruiz (Linares, Jaén, 1998) became interested in mathematics when she was still very young, and went on to study for a degree in that subject at the University of Granada. Although she particularly liked the field of partial differential equations, she did her final degree project in geometry. After completing a master in introduction to research and the applications of mathematics at the Autonomous University of Madrid, she began her PhD at ICMAT under the supervision of Alberto Enciso and Daniel Peralta. This is not her first encounter with research work; she was awarded a JAE introduction to research grant as well as one of the grants for collaboration at Severo Ochoa centres of excellence and María de Maetzu units of excellence - CSIC JAE INTRO SOMdM. In the future, she would like to learn more about mathematical physics.

Pedro Mateos

Alba Dolores García Ruiz has just begun her thesis at ICMAT as part of the European Research Council (ERC) project "Analysis of geometry-driven phenomena in fluid mechanics, PDEs and spectral theory", under the supervision of Alberto Enciso Carrasco and Daniel Peralta Salas. Spectral theory and partial differential equations (PDEs) constitute one of her favourite subjects. "What I like about PDEs is that you have to address each equation in a different way. Every time a new problem appears you have to look for tools again starting from scratch. You have to make the problem you're dealing with your own. You have the feeling that you're retracing your steps over and over, and every time you discover a new detail you were unaware of before."

Mathematics has excited her ever since her time at high school. "What I enjoyed most was arriving at an understanding of mathematics, turning things over in my mind and then explaining them to people. After taking part in a regional mathematical olympiad, I realized that maths was the subject I wanted to study," she says. She was left in no doubt about this when she went to university. "Studying for my degree changed my mindset completely, and replaced it with another one that enabled me to understanding things in a different way."

In the fourth year of her degree course, she received a research grant to study under José Cañizo (University of Granada)'s supervision with whom she worked on solutions to nonlocal PDEs. After that, she was awarded a JAE introduction to research grant that enabled her to attend the JAE school and begin her collaboration with Alberto Enciso. In addition, while studying for a Master in Mathematics and Applications at the Autonomous University of Madrid (UAM), she obtained a JAE SOMdM grant that allowed her to do research work in parallel on inverse localization and global approximation theory, also with Enciso.

She stresses that in her doctorate she is "learning to apply the mathematics I studied in my degree to real-world problems." And although the coronavirus pandemic has affected her work she is still relishing every moment.

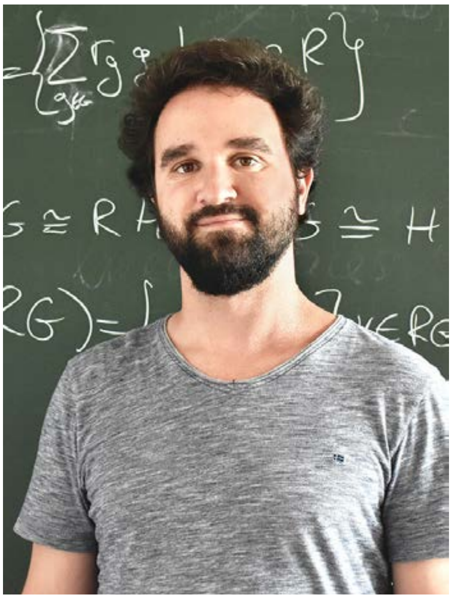
Alba García describes her early experience of research at ICMAT as like "wandering through a place that you don't know but are very eager to discover." She says that it was an awakening for her to move from regarding ICMAT researchers as people who were remote from her and whose work was incomprehensible to people she could talk to at a personal level. "I've been able to speak to the mathematicians I've met in my career as individuals, and I've felt welcomed and supported." The support she's received from Enciso has been vital for her understanding of subjects that previously seemed inaccessible to her. "I'd also like to mention Carolina Vallejo (ICMAT-UC3M) and Eva Gallardo (ICMAT-UCM)", she adds, "two people who have helped me to see certain mathematical ideas in a clearer more straightforward way. They gave courses at the JAE school and made them very accessible, which gave me a very good impression of how everyone works here".

Aside from the world of science, Alba is passionate about books. "I've always found them fulfilling. Before focusing on mathematics I used to write a lot," she says. "Less so now, but I still read a lot of novels and poetry." Lately she has been immersed in the crazy mathematics of Lewis Carroll, whose works "use logic and mathematics in situations arising in daily life, all mixed with literature." She is also enthusiastic about problems that combine numbers and philosophy. "I've spent a lot of time thinking about the paradox of Achilles and the tortoise; I'm fascinated with questions about infinity." Apart from her voluntary work, in her spare time she loves hiking and tourism: "Travelling with a backpack, going from town to town or through the countryside." And for future mathematicians she has a word of advice: "Whoever is thinking about following mathematics as a career, I think they should go ahead, because it's definitely worthwhile."

PROFILE: Leo Margolis (ICMAT-CSIC)

"I want to explore new problems and enjoy mathematics"

Image: Leo Margolis



Leo Margolis joined the ICMAT in August 2021.

Leo Margolis (Samara, Russia, 1988) arrived at ICMAT in August 2021, where he is currently doing a postdoctoral stay to work on algebraic group theory. Before that, he completed his PhD in Stuttgart, Germany, and later obtained postdoctoral contracts at the University of Murcia and the Vrije Universiteit Brussel, in Belgium. Last year, together with the researcher Florian Eisele, he was awarded the Reinhold Baer Prize 2020 for their solution to the Zassenhaus conjecture, a mathematical problem that had remained open for almost fifty years. His main areas of interest are isomorphisms and group rings, about which he has published dozens of papers and has delivered many talks and lectures since 2011.

Pedro Mateos

Every since he was a child, Leo Margolis has felt attracted by mathematics, and as he himself acknowledges he was very good at the subject when at school. "My passion for mathematics began in my childhood, with mathematical games like the Hanoi Tower problem, or another one where you have to take measurements with a coin," he explains. So when he had to make a choice about his professional career, he had no doubt that he wanted to devote himself to mathematics.

"I got involved in research when doing my master final project," he remembers, "but the decisive moment for me were the exercises on group theory I had to tackle at university, before doing my master." He recalls looking at the question sheet and being completely bemused. "Nevertheless, after reading a little about the theory, the penny dropped and everything became clear in my mind. That was my first experience of algebra, and although at the beginning it seemed complicated, I realized that once I had got used to it and understood how it worked, it turned out to be something quite trivial."

His thesis, which he defended at the University of Stuttgart, was supervised by the mathematician Wolfgang Kimmerle and dealt with the so-called group rings. "At that time we were working on vector spaces," says Margolis. "These are systems which, by taking a base, you can extend a multiplication to the entire group ring. The thesis broadened my horizons and helped me to develop new ideas with which I could continue pursuing this avenue." His current interests remain the same; he is still studying these vector structures and how their different objects are mutually connected.

On completion of his postdoc in Belgium, he picked up again some of the ideas he had been working with at the beginning of his thesis. As he explains, "what I first addressed was the Zassenhaus conjecture, which dates from the 1970s." It was during his stay as a postdoctoral researcher at the University of Murcia that he solved the conjecture after finding a counterexample. This result earned him the prestigious Free University of Brussels Prize.

Since he joined ICMAT, Leo has enjoyed a "very pleasant working atmosphere. It's a very nice building and we have a highly active research team that I like very much. I've also got to know a lot of people with whom, although they're working on questions that are different from mine, I've had the opportunity to hold interesting conversations on mathematics." He goes on to say that "I've been involved as much with students as with professors, something that I've found highly motivating, especially after the tough times we've had in recent years." He hopes to continue with his research activity and solve some of the problems he has been working on. "I want to explore new problems, acquire good research experience and generally enjoy mathematics," he says.

He is an enthusiast of literature and film classics in his free time. "I also like learning languages. I was born in Russia and was brought up in Germany, so I've spoken both these languages since I was a child, and English as well, of course. Also, I learned a little Dutch when I was in Belgium, and now I want to continue improving my Spanish."

During his stay in Brussels, he completed a university project that consisted of making an outfit the famous Manneken Pis sculpture based on the Pythagoras theorem. The reason for that? "Nobody had ever made a mathematical costume for this statue before."

His favourite book on mathematics is a work by Bertram Huppert on classical group theory. "It hasn't been translated into German, although the following two volumes are available in English. It's a pity because it's fantastic and contains many important results." While he does not want to look too far ahead in his life, this young researcher might one day go into teaching. "At the moment I just want to finish my work at ICMAT," he says, "and afterwards carry on researching or maybe teaching, whether in Spain or in another country."

SCIENTIFIC REVIEW: Constructing Turing complete Euler flows in dimension 3

Title: "Constructing Turing complete Euler flows in dimension 3".
Authors: Robert Cardona (IRMA-Université de Strasbourg), Eva Miranda (UPC, CRM), Daniel Peralta-Salas (ICMAT-CSIC) and Francisco Presas (ICMAT-CSIC).
Source: *Proceedings of the National Academy of Sciences* vol. 118, no. 19.
Date of publication: May 11 2021.
Link: https://www.pnas.org/content/118/19/e2026818118

Abstract

The computational power of a physical system is measured in terms of its ability to simulate a universal Turing machine. We recall that a Turing machine is a mathematical model of a theoretical device manipulating a set of symbols on a strip of tape with some specific rules; any computer algorithm can be described in terms of Turing machines. During the last decades, several physical processes have been shown to exhibit such Turing completeness, from ray tracing problems in 3D optical systems to neural networks or non-abelian topological quantum field theories. Additionally, Turing completeness of a physical system is intimately related to the undecidability of its evolution, which can be understood as an emergence of complexity in physics totally different from the chaotic behavior.

In contrast, the computational power of fluid dynamics is much less understood. In this direction, Chris Moore asked in 1991 if hydrodynamics is capable of performing computations, i.e., can a fluid ow simulate a universal Turing machine? This question (universality) has been recently analyzed by Terence Tao related to the problem of the regularity of the Euler and Navier-Stokes equations. In particular, Tao speculates on a connection between a potential blow-up of the Navier-Stokes equations, Turing completeness and fluid computation.

In the article under review the authors construct a steady incompressible fluid flow on a Riemannian 3-dimensional sphere that is Turing complete (i.e., it can simulate any Turing machine). This implies the existence of undecidable fluid particle paths, that is, there is no algorithm to ensure that a fluid particle will pass through a certain region of space in finite time. This inability to predict, which is different from that established by chaos theory, can be understood as a new manifestation of the turbulent behavior of fluids.

A steady fluid flow on a Riemannian 3-manifold (M, g) is described by the stationary Euler equations:

$$\begin{cases} \nabla_X X = -\nabla P\,, \\ \operatorname{div} X = 0\,, \end{cases}$$

where X is the velocity field of the fluid (an autonomous vector field on M), P stands for the hydrodynamic pressure. Here $\nabla_X X$ denotes the covariant derivative of X along X.

To construct the aforementioned Turing complete Euler flow, the authors makes use of Beltrami fields, a particularly relevant class of stationary solutions of the Euler equations. Specifically, a divergence-free vector field X on (M, g) is Beltrami if it satisfies the equation

$$\operatorname{curl} X = \lambda X$$

for some constant $\lambda \neq 0$. We recall that the operator curl$X$ is defined as the only vector field on M satisfying

$$i_{\operatorname{curl} X}\mu_g = d\,(i_X g)$$

where μ_g is the Riemannian volume form.

The key aspect of Beltrami fields for the present work is their strong geometric content. Indeed, as first observed by Dennis Sullivan and substantially developed by John Etnyre and Robert Ghrist, any nonvanishing Beltrami field defines a contact structure on M and conversely, the Reeb field of a contact form is a Beltrami field for some adapted Riemannian metric. This Reeb-Beltrami correspondence turns out to be extremely useful to construct steady Euler flows that exhibit complex dynamics: one first constructs Reeb fields exhibiting the desired properties (exploiting the flexibility of the contact and symplectic worlds) and then the aforementioned correspondence yields a Beltrami field for some metric.

To construct a Turing complete Reeb flow, the authors make use of a deep connection between Turing machines and symbolic dynamics. A Turing machine is a mathematical computer manipulating a set of symbols on a tape following some specific rules. It receives, as input data, a sequence of zeros and ones and, after a number of steps, returns a result, also in the form of zeros and ones. More concretely: A Turing machine is defined as $T = (Q, q_0, q_{halt}, \Sigma, \delta)$, where Q is a finite set of states, including an initial state q_0 and a halting state q_{halt}, Σ is the alphabet, and $\delta : (Q \times \Sigma) \to (Q \times \Sigma \times \{-1, 0, 1\})$ is the transition function. The input of a Turing machine is the current state $q \in Q$ and the current tape $t = (t_n)_{n\in\mathbb{Z}} \in \Sigma^{\mathbb{Z}}$. If the current state is q_{halt}, then halt the algorithm and return t as output. Otherwise compute $\delta(q, t_0) = (q', t'_0, \varepsilon)$, replace q with q', t_0 with t'_0 and t by the ε-shifted tape.

A dynamical system on M (e.g. a steady Euler ow) is Turing complete if it can simulate any Turing machine. This means that the halting of any Turing machine with a given input is equivalent to a certain bounded trajectory of the dynamics entering a certain open set of M . This can be understood as a mathematical fluid computer; it takes as input data a point in space, processes it (following the trajectory of the fluid through that point) and its outcome is the next region to which the fluid has moved.

Using the theory of generalized shifts developed by Moore in the early 1990s, the authors construct an area-preserving diffeomorphism of the disk $\varphi : D \to D$, that is Turing complete. More precisely, the restriction of φ to the square Cantor set is conjugate to the transition function of a universal Turing machine (using a canonical assignment between configurations of a Turing machine and points on the square Cantor set).

Then, a suspension technique from symplectic geometry (which in particular also employs Moser's path method) allows the authors to embed the diffeomorphism φ as the Poincaré map on a certain cross section of a Reeb field R defined on the

3-sphere. By construction, R is Turing complete as well and all the computations associated to the universal Turing machine take place on a compact subset of a solid torus $T \subset \mathbb{S}^3$. Accordingly, the aforementioned Reeb-Beltrami correspondence implies the existence of a Riemannian metric g on $\mathbb{S}^3$ for which R satisfies the equation $\text{curl}_g R = R$, so it becomes a Beltrami field on $(\mathbb{S}^3, g)$. Moreover, the metric g is the canonical one in the complement of the solid torus T. This completes the proof of the theorem.

SCIENTIFIC REVIEW: Mixing solutions for the Muskat problem

Title: "Mixing solutions for the Muskat problem".
Authors: Ángel Castro (ICMAT-CSIC), Diego Córdoba (ICMAT-CSIC) and Daniel Faraco (ICMAT-UAM).
Source: *Inventiones mathematicae* volume 226, pages 251–348 (2021).
Date of publication: May 5 2021.
Link: https://link.springer.com/article/10.1007/s00222-021-01045-1

Abstract

The Muskat problem studies the evolution of two incompressible fluids through a porous media, for example, water and fuel through sand. In the unstable regime, in which there is a portion of the heaviest fluid above the lightest one, the classical theory catastrophically fails. However, a long list of experiments and computer simulations predicts that the two fluids mix in a stochastically unpredictable way in a growing in time region called the mixing zone. For years, the problem seem unreachable apart from the case with flat interphases, where there are partial results of Felix Otto and László Székelyhidi.

In this work, Ángel Castro (ICMAT-CSIC), Diego Córdoba (ICMAT-CSIC) and Daniel Faraco (ICMAT-UAM) combine compensated compactness, convex integration, contour dynamics and semiclassical analysis to provide weak solutions to the Muskat problem in the unstable regime.

The convex integration method is linked with Tartar compensated compactness theory. This method codifies the macroscopic behavior of fine microstructures at the mesoscopic level. It studies nonlinear partial differential equations and the underlying phenomena through weak limits. Notice that weak limits are especially suitable for turbulent regimes as they take into account all possible interferences of various length scales as opposed to coarse grained equations or low filtered ones.

The set of weak limits is called the relaxation of the equation. The theory of compensated compactness describes these limits geometrically through directional convexity. The case of incompressible fluids in a porous media case the relaxation is well understood and it is presented in previous works by Córdoba, Faraco and Francisco Gancedo, and Székelyhidi Jr. Recent work by Francisco Mengual shows the explicit relation between the structure of the relaxation and hydrodynamic instabilities).

The convex integration method started with the Nash-Kuiper solution of the isometric immersion problem. It gives a procedure to obtain a solution, starting with a relaxed solution and adding suitably localized oscillating approximate solutions. Reflecting inherent stochasticity of the problem, there are infinitely many subsolutions associated to a relaxed solution, but the observable properties of the solution, the macroscopical behavior, is fully dictated by the relaxed solution.

However, finding subsolutions is also very complicated. In this work, the authors construct a subsolution whose density in a mixing zone is an interpolation of the values of the density of the heavy and light fluid. The mixing zone is described as a point dependent envelope of an evolving in time pseudo interface.

With this ansatz, the pseudo interface is forced to solve a nonlinear and non local equation, which is a double average of the classical Muskat problem. The leading order of such equation evolves according a symbol $p(x, t\backslash k)$, where t is the time. It happens that the symbol degenerates as t tends to zero, so the solution is to treat t as the small parameter analogous to the Planck constant in the semiclassical world, but the symbol is non smooth, which prevents the classical theory to apply. As a matter of fact, in another recent result it is explained the impact of this work in the semiclassical world.

In this paper, the mathematicians provided a method to turn ill posed problems in fluid dynamics into well posed ones, with the additional presence of predicted macroscopical evolutions. As a recent success they have been able to prolong the solution after the Rayleigh-Taylor breakdown obtained by Castro, Córdoba, Fefferman, Gancedo and López-Fernández.

Research Results

Mathematics for tackling the ethical issues of autonomous vehicle

Imagine that given a sudden failure of the brakes in a self-driving car -that is, one in which the driving system is controlled partially or entirely by computer– you were faced with the choice between two possible options; to continue straight ahead, which would result in the death of three pedestrians, or to swerve and hit a wall, which would kill all five occupants of the vehicle. Which one would you choose? And who would make this decision?

New mathematical models proposed by ICMAT (Institute of Mathematical Sciences) researchers Roi Naveiro and David Ríos, together with William N. Caballero (United States Air Force Academy), provide tools for addressing these types of ethical issues concerning autonomous vehicles in a transparent way.

These models take multiple objectives into account for decision-making (performance, length of the journey, passenger safety, pedestrians, the vehicle itself, etc.), the importance of which is determined by the manufacturer. The interest of this approach resides in the fact that it is possible to simulate multiple driving scenarios by using the perspective chosen by the manufacturer for guiding the decisions adopted by the autonomous vehicle. Furthermore, they can also be employed to establish responsibility in the event of an accident.

The work was published in October (2021) in the journal *Decision Analysis*, while a second article on the same subject has been accepted for publication by the journal *Transportation Science*. These results form part of the Trustonomy framework, a project funded by the European H2020 programme, which includes different actors belonging to the automobile industry.

Image: FIA

The introduction of autonomous vehicles is expected to revolutionize the transport of people and goods in the coming years.

The lines of electromagnetic fields can be knotted and maintain this structure

Knotted structures appear in different physical structures; in liquid crystals, in optical phenomena, in fluids and in the lines of electromagnetic field. These knots usually disappear rapidly, and to date only some specific structures that can be maintained over time were known. Now, Benjamin Bode, a Severo Ochoa postdoctoral researcher at ICMAT has demonstrated that the lines of electromagnetic fields can be entangled in all possible forms, and that these structures remain unchanged over time.

This result was published in September (2021) in the journal *Communications in Mathematical Physics* in response to a problem "on which many researchers have been working for years," says Daniel Peralta, an ICMAT researcher who collaborates with Bode.

In recent years, the study of knots in different physical systems has become very popular. Thanks to their approach, the researchers have been able to determine how much control can be exercised in different systems. As Bode explains, "if we can manage to tie any knot, which is the case in electromagnetic fields, we can then exercise a relatively good level of control over the system. If we can only tie simple knots, as seems to be the case of some fields that appear in particle physics, the solutions are somehow very rigid, without much flexibility."

In order to arrive at this achievement, Bode has used a more than 50 year-old result that enables the differential equation problem to be translated to another problem regarding a certain type of complex function. The key has been to unite this result with ideas from another field known as contact topology. This interaction between different areas of mathematics is one of the attractive features of the result. "It's beautiful when a concept in one area arises in another completely different field," says Bode.

Image: ICMAT

Benjamin Bode, a postdoctoral researcher Severo Ochoa at ICMAT, has shown that electromagnetic field lines can be entangled in all possible ways.

Inverse problems in Africa

Faguèye Ndiaye (Cheikh Anta Diop University, Dakar, Senegal) has just completed her stay at ICMAT as part of the 6th edition of the Women for Africa "Science by women" programme. She has been collaborating with the AGAPI research group (Analysis and Geometry with Applications to Inverse Problems), headed by Daniel Faraco, a professor at the Autonomous University of Madrid (UAM) and a member of ICMAT.

The Institute has been collaborating with this initiative since it first began. It is thanks to this project that senior women scientists from African countries have been working at the Severo Ochoa centres of excellence involved over a six-month period in this research under-taking. The main aim is to provide a leading role for women scientists from this continent in transforming the economy to one based on knowledge and innovation in their respective countries.

Image: African Women in Mathematics Association

Faguèye Ndiaye (Cheikh Anta Diop de Dakar University, Senegal) has worked in the field of inverse problems during her stay at the ICMAT.

Awards

David Pérez García, the Royal Academy of Sciences Ramón y Cajal Medal winner

On the third occasion of the distinction, the Royal Spanish Academy of Exact, Physical and Natural Sciences (RAC) has awarded the Ramón y Cajal Medal to David Pérez García, ICMAT researcher and a professor at the Complutense University of Madrid (UCM). This award has been conferred since 2017 on researchers younger than 50 for their scientific achievements, and this is the first time a mathematician has been distinguished with the medal.

Pérez García's research is focused on the applications of mathematics to quantum information theory. He leads the ICMAT Ignacio Cirac Laboratory on this topic and, also, heads the Mathematics and Quantum Information research group at the UCM. Pérez García is the author of more than 90 papers published in high impact journals, *Nature* and *PNAS* among them. He is also the principal researcher of projects that amount to a total of more than five million euros, among which is an ERC Consolidator Grant that he obtained in 2015.

Image: David Pérez García

David Pérez García, a ICMAT researcher and professor at the Complutense University of Madrid, works on quantum information theory.

Antonio Córdoba receives the Royal Spanish Mathematical Society medal

Every year, the Royal Spanish Mathematical Society (RSME) awards its medals to "people outstanding for their relevant, exceptional and continual contribution to any area of mathematical endeavour." Among the three distinguished with an award in 2021 is Antonio Córdoba, ex-director (2016 - 2019) and now an associate member of ICMAT, as well as professor emeritus of mathematical analysis at the UAM. Together with him, Tomás Recio, a professor at the University of Cantabria, and Olga Gil Medrano, a professor at the University of Valencia, were also awarded a medal.

Córdoba has conducted his research in different areas, ranging from number theory to the study of harmonic analysis, partial differential equations and mathematical physics. His work has had a broad impact and has received international recognition, with publications in journals such as *Annals of Mathematics, Inventiones Matematicae, Communications in Mathematical Physics, Proceedings of the National Academy of Sciences* and *Duke Mathematical Journal*. He has been the recipient of distinguished prizes, among which is the Julio Rey Pastor Mathematics and Communication Sciences National Prize, which he was awarded in 2011.

Image: ICMAT

Córdoba received in 2011 the Julio Rey Pastor National Prize for Mathematics and Communication Sciences.

Scientific activities

Charles Fefferman inaugurates the season of ICMAT Distinguished Lectures

On October 15th (2021), the latest cycle of ICMAT Distinguished Lectures got under way, a series of talks given by leading figures of international standing in mathematics. The opening lecture was delivered by Charles Fefferman, a professor at Princeton University, a Fields Medal winner and director of one of the ICMAT Laboratories.

Fefferman's research work has dealt with several different areas: harmonic analysis, partial differential equations, Fourier analysis, mathematical physics, fluid dynamics, neural networks and differential geometry, among others. An example of this broad range of interests were the two subjects he addressed at the Institute; interpolation of data with smooth functions and mathematical problems in graphene.

The ICMAT Distinguished Lectures consist of a new series of talks by outstanding high-impact male and female mathematicians who will share their interests with the community as a whole. Two such events will be held every year, the next lecture being given on April 1st 2022 by Claire Voisin, a CNRS researcher at the Institut de Mathématiques de Jussieu-Paris Rive Gauche.

Image: ICMAT

Charles Fefferman, in his conference organized by the ICMAT.

The first joint colloquium between ICMAT and the Autonomous, Complutense and Carlos III Madrid universities

Concha Bielza, a professor at the Polytechnic University of Madrid and a specialist in automatic learning and Bayesian networks, was the first speaker at the ICMAT-UAM-UC3M-UCM Joint Mathematical Colloquium. This event, entitled "Bayesian Networks and Temporal Data", was held on October 1st at ICMAT and was also streamed online.

This gathering signalled a new type of colloquium conducted in collaboration between ICMAT and the three Madrid universities (Autonomous, Complutense and Carlos III) of which the Institute forms part. It thereby replaces the former UAM-ICMAT Colloquium and the ICMAT-UCM Colloquium, and will consist of an itinerant activity that will be held at the four collaborating institutions.

The second colloquium took place on November 30th (2021) and was imparted by Eva Miranda, a professor at the Universitat Politècnica de Catalunya. In her talk, entitled "From Euler equations to Turing machines via contact geometry", she presented the recent progress she has made in fluid dynamics using Turing machines. The meeting was held at the UCM Faculty of Mathematical Sciences.

Image: UPM

Bielza has participated in research projects on medicine, neuroscience, biology, agriculture, reservoir management or even predicting the price of the electricity market.

The ICMAT Hitchin-Ngô Laboratory gets under way

The Nigel Hitchin – Ngô Bảo Châu Laboratory, one of the four new ICMAT Laboratories funded by the Severo Ochoa Programme for Excellence, began its activities in June 2021 with the Opening of the Hitchin-Ngô ICMAT Laboratory event, which was held online.

Steven Bradlow (University of Illinois at Urbana-Champaign, USA) occupied the chair and scientific talks were given by the two researchers after whom the Laboratory is named, Nigel Hitchin (University of Oxford, UK) and Ngô Bảo Châu (University of Chicago, USA), together with coordinator Oscar García-Prada (ICMAT).

Outreach activities

The ICMAT, present at the Science Week and Researchers' Night

The ICMAT participated in the programme of activities organized by the Consejo Superior de Investigaciones Científicas (CSIC) for the Science and Technology Week. On this occasion the Institute scheduled two talks held on November 11th: "Mathematics for deciphering the mysteries of fluids", given by Diego Córdoba (ICMAT-CSIC) and aimed at baccalaureate students, and "Autonomous vehicles, ethics and mathematics", delivered by David Ríos (ICMAT-CSIC) and Roi Naveiro (ICMAT-CSIC) and intended for the general public, in collaboration with the RAC. Additionally, the UAM+CSIC Intercenter Equality Commission, to which the ICMAT belongs, scheduled "Escape-Road: In search of Nobel and non-Nobel women scientists" at the Eugenio Trías Library in Madrid between November 10th-14th.

Furthermore, together with other centres of the UAM+CSIC Intercenter Equality Commission, the ICMAT also took part in the 2021 European Researchers' Night on September 24th. On this occasion, the centres organized a scientific fair, "The science train stops in Atocha: the CSIC Night", in the tropical garden of the Madrid Puerta de Atocha railway station. The ICMAT researchers David Martín de Diego and Alexandre Anahory ran a workshop called "Learn mathematics by playing with the ICMAT", during which they proposed various games whose strategy could be analyzed by means of mathematical concepts belonging to graph theory and topology.

Image: ICMAT

David Martín de Diego and Alexandre Anahory, ICMAT researchers, participated in the Researchers' Night 2021.

Antonio Córdoba and Amie Wilkinson, at Mathematics at the Residencia

On September 28th, the second meeting in 2021 of the series of outreach talks 'Mathematics at the Residencia' was held. The talk was entitled "By the rotation of a needle," which was given by Antonio Córdoba, who on this occasion presented the Kakeya problem, a question at the heart of modern mathematical analysis proposed in the early 20th century that has attracted the interest of Fields Medal winners such as Jean Bourgain and Terence Tao, and to which Córdoba himself has made important contributions.

The first meeting in this year's series took place in May and the talk was given by Amie Wilkinson, a professor at the University of Chicago and an expert in dynamical systems. This researcher devoted her talk to recurrence, a key concept in dynamics and the mathematical equivalent of *déjà vu*. "Recurrence is a simple but powerful concept that can be used to respond to a surprising variety of questions, ranging from how two gases mix in a box to the deep properties of prime numbers and the discovery of exoplanets in nearby solar systems," explains Amie, a mathematician who was awarded the Satter Prize in Mathematics in 2011 and the Levi L. Conant Prize in 2020.

Marta Macho-Stadler publishes a book on mathematics and literature

Mathematics can be found in different literary texts, such as in Sherlock Homes stories and books by Jules Verne as well as in works by writers belonging to the French literary group known as "OuLiPo" (*Ouvroir de Littérature Potentielle*). Marta Macho-Stadler, a professor at the Universidad del País Vasco (UPV/EHU) and a disseminator of mathematics, explores the links between maths and literature in her book *Mathematics and Literature*. This book has recently appeared as part of the collection "Miradas Matemáticas", a joint publishing venture between ICMAT, the Spanish Federation of Societies of Mathematics Teachers (FESPM) and the publisher Los Libros de la Catarata.

CALENDAR

ICMAT Scientific activities

- **Workshop on Banach spaces and Banach lattices II**
  Date: May 9-13, 2022
- **Graduate school on Geometric Group Theory and Low Dimensional Topology**
  Date: May 16-27, 2022
- **Geometric Aspects of the Swampland 2022**
  Date: May 23-26 and June 8-10, 2022
- **11th International Conference on Harmonic Analysis and Partial Differential Equations**
  Date: June 6-10, 2022
- **Poisson 2022 Conference**
  Date: July 25-29, 2022
- **Moduli spaces and geometric structures. Conference in honour of Oscar García-Prada on the occasion of his 60th birthday**
  Date: September 12-16, 2022
- **Intercity Seminar on Arakelov Geometry 2022**
  Date: September 12-16, 2022

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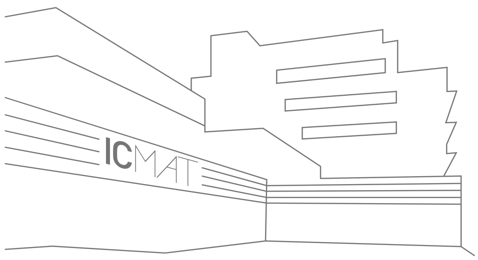

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