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EDITORIAL

The surprising usefulness of knowledge generated merely by intellectual curiosity

Eighty years ago, Abraham Flexner published a timeless <u>essay</u> in Harper's Magazine, the title of which, "The Usefulness of Useless Knowledge", we have paraphrased for the headline in this article. In 1929, Flexner, who was known in the United States for his publications on medical studies, contacted Caroline and Louis Bamberger, a brother and sister who had amassed an enormous fortune with the sale of their chain of department stores in New Jersey just before the Wall Street crash, and the Bamberger siblings were thinking about investing part of the proceeds in a new Faculty of Medicine in Newark.

However, Flexner proposed a much more ambitious project to the Bambergers: to create in the USA an institute devoted not only to the education of degree students but also to research. He managed to persuade them to invest five million dollars (at the value at that time) in what came to be known as the Institute for Advanced Study (IAS). Ultimately, it came to be built in Princeton rather than in Newark, together with a large university consisting of an outstanding intellectual community, libraries and other facilities.

The IAS was officially created on May 20th, 1930, with Flexner appointed as its first director by the Bambergers. In the <u>memo-randum</u> he presented to the Board of Trustees, Flexner wrote as follows: "The Institute should be small and plastic (i.e. flexible). It should also be a paradise where scientists can look at the world and its phenomena as a laboratory, without being disturbed by the immediate concerns of current events; (....) Its professors should enjoy full intellectual freedom and be completely relieved of all administrative responsibilities and bureaucratic tasks".

Every course at the IAS consists of a new mixture of ingenuity from its temporary members, ranging from young doctoral students to distinguished visiting professors, who typically form part of the Institute for a year, a period that may extend up to five years, and who may return on future occasions throughout their careers. The IAS currently consists of four schools (Mathematics, Natural Sciences (Theoretical Physics), History and Social Sciences), Mathematics and Theoretical Physics being those that initially existed. The most distinguished professors during this initial period included Albert Einstein, Kurt Gödel, Hermann Weyl and John von Neumann, among others.

My first visit to the IAS took place when I joined it as an Assistant Professor at Princeton University in 1974. I subsequently became a fellow/member of the IAS in the 1989-90 academic year and then as a visiting professor on several occasions. The Institute remains in my memory as a magical place where I was able to collaborate with the most creative mathematicians of the moment and converse with mythical figures in my mathematical training such as Arne Behrling, Atlee Selberg and André Weil. I also got to meet, although not to talk to, Kurt Gödel himself in person, whose impact on my mathematical career has been of great importance.

I was also able to get to know at first hand the utopia conceived and promoted by Flexner. The centre was truly a paradise for intellectuals, a space of freedom in which scientists could pursue their ideas undisturbed by the madding crowd. After the time they spent there, they were not obliged to give an account of themselves to anyone. Their work did not necessarily have to be focused on the solving of great problems or changing the world, although in many cases their achievements there did in fact do just that. The design of the first modern computer (the ENIAC Project) by von Neumann is perhaps the greatest example of the astonishing usefulness of thought motivated solely by intellectual curiosity that Flexner had in mind. Some years after the creation of the Institute, Flexner expressed his reflections in the essay he published in Harper's Magazine, in which he demonstrated the enormous usefulness that research motivated by curiosity can ultimately have. In what follows, we give some representative extracts from the essay, translated into Spanish.

Antonio Córdoba, ICMAT director between 2016 and 2019, UAM Professor Emeritus

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The Usefulness of Useless Knowledge

By Abraham Flexner • Published in 1939

Is it not a curious fact that in a world steeped in irrational hatreds which threaten civilization itself, men and women-old and young—detach themselves wholly or partly from the angry current of daily life to devote themselves to the cultivation of beauty, to the extension of knowledge, to the cure of disease, to the amelioration of suffering, just as though fanatics were not simultaneously engaged in spreading pain, ugliness, and suffering? The world has always been a sorry and confused sort of place—yet poets and artists and scientists have ignored the factors that would, if attended to, paralyze them. From a practical point of view, intellectual and spiritual life is, on the surface, a useless form of activity, in which men indulge because they procure for themselves greater satisfactions than are otherwise obtainable. In this paper I shall concern myself with the question of the extent to which the pursuit of these useless satisfactions proves unexpectedly the source from which undreamed-of utility is derived.

We hear it said with tiresome iteration that ours is a materialistic age, the main concern of which should be the wider distribution of material goods and worldly opportunities.



Institute for Advanced Study (Princeton University)

The justified outcry of those who through no fault of their own are deprived of opportunity and a fair share of worldly goods therefore diverts an increasing number of students from the studies which their fathers pursued to the equally important and no less urgent study of social, economic, and governmental problems. I have no quarrel with this tendency. The world in which we live is the only world about which our senses can testify. Unless it is made a better world, a fairer world, millions will continue to go to their graves silent, saddened, and embittered. I have myself spent many years pleading that our schools should become more acutely aware of the world in which their pupils and students are destined to pass their lives. Now I sometimes wonder whether that current has not become too strong and whether there would be sufficient opportunity for a full life if the world were emptied of some of the useless things that give it spiritual significance; in other words, whether our conception of what is useful may not have become too narrow to be adequate to the roaming and capricious possibilities of the human spirit.

We may look at this question from two points of view: the scientific and the humanistic or spiritual. Let us take the scientific first. I recall a conversation which I had some years ago with Mr. George Eastman on the subject of use. Mr. Eastman, a wise and gentle farseeing man, gifted with taste in music and art, had been saying to me that he meant to devote his vast fortune to the promotion of education in useful subjects. I ventured to ask him whom he regarded as the most useful worker in science in the world. He replied instantaneously: "Marconi." I surprised him by saying, "Whatever pleasure we derive from the radio or however wireless and the radio may have added to human life, Marconi's share was practically negligible."

I shall not forget his astonishment on this occasion. He asked me to explain. I replied to him somewhat as follows: "Mr. Eastman, Marconi was inevitable. The real credit for everything that has been done in the field of wireless belongs, as far as such fundamental credit can be definitely assigned to anyone, to Professor Clerk Maxwell, who in 1865 carried out certain abstruse and remote calculations in the field of magnetism and electricity. Maxwell reproduced his abstract equations in a treatise published in 1873. At the next meeting of the British Association Professor H. S. Smith of Oxford declared that 'no mathematician can turn over the pages of these volumes without realizing that they contain a theory which has already added largely to the methods and resources of pure mathematics.' Other discoveries supplemented Maxwell's theoretical work during the next fifteen years. Finally in 1887 and 1888 the scientific problem still remaining—the detection and demonstration of the electromagnetic waves which are the carriers of wireless signals-was solved by Heinrich Hertz, a worker in Helmholtz's laboratory in Berlin. Neither Maxwell nor Hertz had any concern about the utility of their work; no such thought ever entered their minds. They had no practical objective. The inventor in the legal sense was of course Marconi, but what did Marconi invent? Merely the last technical detail, mainly the now obsolete receiving device called coherer, almost universally discarded."

Hertz and Maxwell could invent nothing, but it was their useless theoretical work which was seized upon by a clever technician and which has created new means for communication, utility, and amusement by which men whose merits are relatively slight have obtained fame and earned millions. Who were the useful men? Not Marconi, but Clerk Maxwell and Heinrich Hertz. Hertz and Maxwell were geniuses without thought of use. Marconi was a clever inventor with no thought but use.

The mention of Hertz's name recalled to Mr. Eastman the Hertzian waves, and I suggested that he might ask the physicists of the University of Rochester precisely what Hertz and Maxwell had done; but one thing I said he could be sure of, namely, that they had done their work without thought of use and that throughout the whole history of science most of the really great discoveries which had ultimately proved to be beneficial to mankind had been made by men and women who were driven not by the desire to be useful but merely the desire to satisfy their curiosity.

"Curiosity?" asked Mr. Eastman.

"Yes," I replied, "curiosity, which mayor may not eventuate in something useful, is probably the outstanding characteristic of modern thinking. It is not new. It goes back to Galileo, Bacon, and to Sir Isaac Newton, and it must be absolutely unhampered. Institutions of learning should be devoted to the cultivation of curiosity and the less they are deflected by considerations of immediacy of application, the more likely they are to contribute not only to human welfare but to the equally important satisfaction of intellectual interest which may indeed be said to have become the ruling passion of intellectual life in modern times."

What is true of Heinrich Hertz working quietly and unnoticed in a corner of Helmholtz's laboratory in the later years of the nineteenth century may be said of scientists and mathematicians the world over for several centuries past. [...]

In the domain of higher mathematics almost innumerable instances can be cited. For example, the most abstruse mathematical work of the eighteenth and nineteenth centuries was the "NonEuclidian Geometry." Its inventor, Gauss, though recognized by his contemporaries as a distinguished mathematician, did not dare to publish his work on "Non-Euclidian Geometry" for a quarter of a century. As a matter of fact, the theory of relativity itself with all its infinite practical bearings would have been utterly impossible without the work which Gauss did at Gottingen.

Again, what is known now as "group theory" was an abstract and inapplicable mathematical theory. It was developed by men who were curious and whose curiosity and puttering led them into strange paths; but "group theory" is to-day the basis of the quantum theory of spectroscopy, which is in daily use by people who have no idea as to how it came about.

The whole calculus of probability was discovered by mathematicians whose real interest was the rationalization of gambling. It has failed of the practical purpose at which they aimed, but it has furnished a scientific basis for all types of insurance, and vast stretches of nineteenth century physics are based upon it. (...)

I am not for a moment suggesting that everything that goes on in laboratories will ultimately turn to some unexpected practical use or that an ultimate practical use isits actual justification. Much more am I pleading for the abolition of the word "use," and for the freeing of the human spirit. To be sure, we shall thus free some harmless cranks. To be sure, we shall thus waste some precious dollars. But what is infinitely more important is that we shall be striking the shackles off the human mind and setting it free for the adventures which in our own day have, on the one hand, taken Hale and Rutherford and Einstein and their peers millions upon millions of miles into the uttermost realms of space and, on the other, loosed the boundless energy imprisoned in the atom. What Rutherford and others like Bohr and Millikan have done out of sheer curiosity in the effort to understand the construction of the atom has released forces which may transform human life; but this ultimate and unforeseen and unpredictable practical result is not offered as a justification for Rutherford or Einstein or Millikan or Bohr or any of their peers. Let them alone. No educational administrator can possibly direct the channels in which these or other men shall work. The waste, I admit again, looks prodigious. It is not really so. All the waste that could be summed up in developing the science of bacteriology is as nothing compared to the advantages which have accrued from the discoveries of Pasteur, Koch, Ehrlich, Theobald Smith, and scores of others-advantages that could never have accrued if the idea of possible use had permeated their . minds. These great artists-for such are scientists and bacteriologists-disseminated the spirit which prevailed in laboratories in which they were simply following the line of their own natural curiosity.

I am not criticising institutions like schoolsof engineering or law in which the usefulness motive necessarily predominates. Not infrequently the tables are turned, and practical difficulties encountered in industry or in laboratories stimulate theoretical inquiries which may or may not solve the problems by which they were suggested, but may also open up new vistas, uselessat the moment, but pregnant with future achievements, practical and theoretical.

With the rapid accumulation of "useless" or theoretic knowledge a situation has been created in which it has become increasingly possible to attack practical problems in a scientific spirit. Not only inventors, but "pure" scientists have indulged in this sport. I have mentioned Marconi, an inventor, who, while a benefactor to the human race, as a matter of fact merely "picked other men's brains." Edison belongs to the same category. Pasteur was different. He was a great scientist; but he was not averse to attacking practical problems-such as the condition of French grapevines or the problems of beer-brewing-and not only solving the immediate difficulty, but also wresting from the practical problem some far-reaching theoretic conclusion, "useless" at the moment, but likely in some unforeseen manner to be "useful" later. Ehrlich, fundamentally speculative in his curiosity, turned fiercely upon the problem of syphilis and doggedly pursued it until a solution of immediate practical use-the discovery of salvarsan -was found. The discoveries of insulin by Banting for use in diabetes and of liver extract by Minot and Whipple for use in pernicious anemia belong in the same category: both were made by thoroughly scientific men, who realized that much "useless" knowledge had been piled up by men unconcerned with its practical bearings, but that the time was now ripe to raise practical questions in a scientific manner.

Thus it becomes obvious that one must be wary in attributing scientific discovery wholly to anyone person. Almost every discovery has a long and precarious history. Someone finds a bit here, another a bit there. A third step succeeds later and thus onward till a genius pieces the bits together and makes the decisive contribution. Science, like the Mississippi, begins in a tiny



rivulet in the distant forest. Gradually other streams swell its volume. And the roaring river that bursts the dikes is formed from countless sources.

I cannot deal with this aspect exhaustively, but I may in passing say this: over a period of one or two hundred years the contributions of professional schools to their respective activities will probably be found to lie, not so much in the training of men who may to-morrow become practical engineers or practical lawyers or practical doctors, but rather in the fact that even in the pursuit of strictly practical aims an enormous amount of apparently useless activity goes on. Out of this useless activity there come discoveries which may well prove of infinitely more importance to the human mind and to the human spirit than the accomplishment of the useful ends for which the schools were founded. The considerations upon which I have touched emphasize-if emphasis were needed-the overwhelming importance of spiritual and intellectual freedom. I have spoken of experimental science; I have spoken of mathematics; but what I say is equally true ofmusic and art and of every other expression of the untrammeled human spirit. The mere fact that they bring satisfaction to an individual soul bent upon its own purification and elevation is all the justification that they need. And in justifying these without any reference whatsoever, implied or actual, to usefulness we justify colleges, universities, and institutes of research. An institution which sets free successive generations of human souls is amply justified whether or not this graduate or that makes a so-called useful contribution to human knowledge. A poem, a symphony, a painting, a mathematical truth, a new scientific fact, all bear in themselves all the justification that universities, colleges, and institutes of research need or require.

The subject which I am discussing has at this moment a peculiar poignancy. In certain large areas-Germany and Italy especially-the effort is now being made to clamp down the freedom of the human spirit. Universities have been so reorganized that they have become tools of those who believe in a special political, economic, or racial creed. Now and then a thoughtless individual in one of the few democracies left in this world will even question the fundamental importance of absolutely untrammeled academic freedom. The real enemy of the human race is not the fearless and irresponsible thinker, be he right or wrong. The real enemy is the man who tries to mold the human spirit so that it will not dare to spread its wings, as its wings were once spread in Italy and Germany, as well as in Great Britain and the United States.

This is not a new idea. It was the idea which animated von Humboldt when, in the hour of Germany's conquest by Napoleon, he conceived and founded the University of Berlin. It is the idea which animated President Gilman in the founding of the Johns Hopkins University, after which every university in this country has sought in greater or less degree to remake itself. It is the idea to which every individual who values his immortal soul will be true whatever the personal consequences to himself. Justification of spiritual freedom goes, however, much farther than originality whether in the realm of science or humanism, for it implies tolerance throughout the range of human dissimilarities. In the face of the history of the human race what can be more silly or ridiculous than likes or dislikes founded upon race or religion? Does humanity want symphonies and paintings and profound scientific truth, or does it want Christian symphonies, Christian paintings, Christian science, or Jewish symphonies, Jewish paintings, Jewish science, or Mohammedan or Egyptian or J apanese or Chinese or American or German or Russian or Communist or Conservative contributions to and expressions of the infinite richness of the human soul?

Among the most striking and immediate consequences of foreign intolerance I may, I think, fairly cite the rapid development of the Institute for Advanced Study, established by Mr. Louis Bamberger and his sister, Mrs. Felix Fuld, at Princeton, New Jersey. The founding of the Institute was suggested in 1930. It was located at Princeton partly because of the founders' attachment to the State of New Jersey, but, in so far as my judgment was concerned, because Princeton had a small graduate school of high quality with which the most intimate cooperation was feasible. To Princeton University the Institute owes a debt that can never be fully appreciated. The work of the Institute with a considerable portion of its personnel began in 1933. On its faculty are eminent American scholars-Veblen, Alexander, and Morse, among the mathematicians; Meritt, Lowe, and Miss Goldman among the humanists; Stewart, Riefl.er, Warren, Earle, and Mitrany among the publicists and economists. And to these should be added scholars and scientists of equal caliber already assembled in Princeton University, Princeton's library, and its laboratories. But the Institute for Advanced Study is indebted to Hitler for Einstein, Weyl, and von Neumann in mathematics; for Herzfeld and Panofsky in the field of humanistic studies, and for a host of younger men who during the past six years have come under the influence of this distinguished group and are already adding to the strength of American scholarship in every section of the land. (...)

No routine is followed; no lines are drawn between professors, members, or visitors. Princeton students and professors and Institute members and professors mingle so freely as to be indistinguishable. Learning as such is cultivated. The results to the individual and to society are left to take care of themselves. No faculty meetings are held; no committees exist. Thus men with ideas enjoy conditions favorable to reflection and to conference. A mathematician may cultivate mathematics without distraction; so may a humanist in his field, an economist or a student of politics in his. Administration has been minimized in extent and importance. Men without ideas, without power of concentration on ideas, would not be at home in the Institute.

I can perhaps make this point clearer by citing briefly a few illustrations. A stipend was awarded to enable a Harvard professor to come to Princeton: he wrote asking,

"What are my duties?"

I replied: "You have .no duties-only opportunities."

(...)

We make ourselves no promises, but we cherish the hope that the unobstructed pursuit of useless knowledge will prove to have consequences in the future as in the past. Not for a moment, however, do we defend the Institute on that ground. It exists as a paradise for scholars who, like poets and musicians, have won the right to do as they please and who accomplish most when enabled to do so.

THEMATIC QUARTER: Algebra and quantum information

The mathematics that shape the quantum information theory

In the 1930s, the mathematician John von Neumann developed the theory of operator algebras in order to solve certain statistical mechanics problems he was working on relating to spectral theory in infinite dimensional Hilbert spaces. Since that time, this theory has enabled many other fields of mathematics to be connected and has become a natural language for the mathematical analysis of processes in quantum physics. In particular, it is important in the study of some aspects of quantum information theory. Barbara Kraus, a physicist at the University of Innsbruck (Austria), says that "mathematics is both the language and the tool. It is necessary for showing that things work, for predicting phenomena, for understanding the theorem better and for arriving at fundamental problems". In 2019, the ICMAT hosted a thematic trimester on this subject, which enabled us to talk to some of the many experts who passed through the centre, including Kraus herself.

Ágata A. Timón G-Longoria. Between March and June, 2019, the thematic programme "<u>Operator Algebras, Groups and Applica-</u><u>tions to Quantum Information</u>" took place at the ICMAT. It included two research schools (with six ten-hour courses given by leaders in the field), two workshops, a conference, a visitors' programme, a series of weekly seminars, and an inaugural colloquium given by Joachim Cuntz (*WWU-Münster*), one of the leading lights in the field of operator algebras. In total, more than 140 researchers from more than 15 different countries came to the Institute.

The relation between the fields named in the title of the programme (operator algebras, groups and quantum information) is currently proving to be especially fruitful, and the development of quantum computers is one of its great technological promises. Quantum computers bring together new and revolutionary data processing tools based on the particularities of quantum physics.

The advantages and limitations of codification in quantum information systems have been explored since the early 1990s. "Quantum physics opens up many possibilities that are not available in classical physics. In the context of data processing, it is possi-



Joachim Cuntz (WWU-Münster), one of the leading lights in the field of operator algebras, gave the inaugural colloquium.

ble to conduct certain processes that either take much longer to do or cannot be done at all in the classical way," says Barbara Kraus, a researcher with the *Entanglement theory and quantum information theory* group at the University of Innsbruck (Austria) and one of the leading participants in the thematic programme.

For its part, operator algebra theory emerged in the 1930s at the hands of the famous mathematician John von Neumann. "For me, the most remarkable thing is that it connects with many other fields of mathematics. In its origins, the theory is linked to mathematical physics, in particular, to statistical mechanics. However, it has gone on progressing and currently is much more closely related to group theory and dynamical systems. The fact that it now has so many connections with quantum information theory makes it a very fruitful subject", says Stefaan Vaes, a professor at the K.U. Leuven (Belgium) and another participant in the programme.

While for a long time these two theories - operator algebras and quantum theory - were developed independently of each other, in the last decade operator spaces and operator systems have proved to be very useful tools for tackling different problems in quantum information, and have connected both fields together again. "In a certain sense, the language of quantum information theory is also the language of operator algebras", says Vaes. This relation can be traced back to von Neumann, to the way of expressing guantum mechanics, where the observables (physical quantities that we can measure in a system) are operators in a Hilbert space, which therefore constitute a von Neumann algebra. Vaes goes on to say that "All kinds of methods were devised to address these operators. Furthermore, classical probability theory seeks to understand how much information can be contained in different channels. When dealing with quantum channels, we have to employ new tools, since we scarcely understand the basic principles. The language for expressing the transmission of information along these channels is the language of operator algebras".

In recent years, different results have been obtained thanks to this interaction: the use of operator algebra techniques in quantum information has been vital for solving some of the open problems in the Bell inequality theorem, non-local games and the theory



The thematic programme had a huge variety of activities. In the picture, participants at the second conference

of quantum channels. Inversely, the use of quantum information techniques has been crucial for the progress made in problems concerning noncommutative Grothendieck inequalities, with the aim of finding new relations between noncommutative Lp spaces in order to gain new perspectives on the so-called Connes immersion problem, one of the great open problems in this field.

The *Connes immersion problem* is related to many other areas of mathematics. The original formulation seeks to understand whether all the algebras of *type II.1* (very complex objects whose classification is still open) are really contained in the *ultra-power* of the smallest of these algebras. "This conjecture seems very technical," says researcher Vaes, "but what it really means is that, in some way, all these von Neumann algebras have a certain finite dimensional approximation." In turn, this approach enabled the problem to be related with group theory, thereby connecting it with the third pillar on which the conference is based. In particular, the Connes immersion problem is equivalent to the construction of certain norms in spaces related to discrete groups.

These promising advances have made this field a focus of interest for specialists in different areas, among whom are found IC-MAT researchers such as Fernando Lledó (UC3M-ICMAT), Diego Martínez (UC3M-ICMAT) and Carlos Palazuelos (UCM-ICMAT). These three, together with Cécilia Lancien (Université de Toulouse) and Julio de Vicente (UC3M), were the organizers of the "Operator Algebras, Groups and Applications to Quantum Information" thematic programme.

The main goal of this thematic programme is to pursue the study of operator algebras and their relation with group theory and quantum information. To this end, many leaders in the field came together over these months, alongside PhD students and young researchers in mathematics and mathematical physics, with interests in apparently disparate fields such as noncommutative algebra, dynamical systems, groups, harmonic analysis, topology and quantum theory. In this way, said the organizers of the programme, "they hoped to stimulate the already fertile interaction between these fields of mathematics and the relatively young field of quantum information".

Were these hopes realized? According to Fernando Lledó: "The questions, collaborations and debates arising from the researchers in different fields regarding common open problems show that the program has been a success". There is no doubt about the interest it aroused. Among the many other young researchers who attended from all over the world, fifteen students who are currently doing courses in North American universities were able to attend the programme thanks to funding from the National Science Foundation (USA).

Programme of activities

The trimester kicked off with a school devoted to the classification of operator algebras and the average rating of groups and spaces. As part of this school, Joachim Cuntz, one of the leading researchers in this area, gave the inaugural colloquium "The ring of integers and C*-algebras" on Wednesday, March 12th. In his talk he spoke about C*-algebras, objects that arise naturally from the basic structures formed by sets of natural numbers and integers with their operations of addition and multiplication, but which give rise to surprisingly rich structures.

"C*algebras arise wherever there is any topological structure", explains Vaes. In the same way, von Neumann algebras appear naturally in any area of mathematics where measure spaces appear. As Vaes goes on to say: "What interests me is the way in which they arise in group theory and ergodic theory. All the groups are codified in complex form algebras, and all the actions of groups and measurable spaces are too. I'm conducting research in order to understand which ones are isomorphs and which aren't, and also to develop new tools on the way".

The field has grown enormously since the 1930s, and key results such as the <u>classification of injective factors</u>, by Alain Connes and Uffe Haagerup, have been obtained. "It is a complete classification theorem. It answers questions formulated by von Neumann in the 1930s", says Vaes. "Alain Connes solved it for all cases except one in 1976, for which he was awarded the Fields Medal. Ten years later, in the mid-1980s, Uffe Haagerup solved the remaining case. At the moment it is a question of establishing similar results for nuclear C*-algebras". This was the topic of one of the courses at the school.

The first conference of the trimester was held between March 18th and 22nd; the second school between May 6th and 10th, and the second conference between May 13th and 17th. The crowning event of the trimester was the International Conference held between June 17th and 22nd. The scientific committee of the trimester included leading international experts in the field: Pere Ara (Autonomous University of Barcelona), Marius Junge (University of Illinois at Urbana-Champaign, USA), David Kerr (Texas A&M University, College Station, USA), Fernando Lledó (Carlos III University of Madrid and the ICMAT); Francesc Perera (Autonomous University of Barcelona); Andreas Winter (Autonomous University of Barcelona) and Mikael Rørdam (University of Copenhagen, Denmark).



The thematic programme concluded with the International Congress, held between June $17^{\rm th}$ and June $22^{\rm nd}$

"Von Neumann algebras appear in any field of mathematics where there is some measure"

Interview with Stefaan Vaes (K.U. Leuven), a participant at the ICMAT "Operator Algebras, Groups and Applications to Quantum Information" thematic programme

Stefaan Vaes is a professor at the K.U. Leuven (Leuven, Belgium) Department of Mathematics. His research work, which is focused on Von Neumann algebras and quantum groups, has been recognized by the European Research Council (ERC) with a Starting Grant (2008-2013) and an Consolidator Grant (2014-2019) project. He is a member of the Belgian Royal Academy of Sciences and a fellow of the American Mathematical Society. He was one of the guest speakers at the International Congress of Mathematicians that was held in Hyderabad (India) in 2010. In March 2019 he gave a course entitled "Type III factors, free Araki-Woods factors and their (non-)classification" as part of the first school of the "Operator Algebras, Groups and Applications to Quantum Information" thematic programme. We met him during a break in the activities and spoke to him about his work.

Ágata A. Timón García-Longoria.

Q: You've visited the ICMAT several times before. What's your relationship with the Institute?

A: My first visit here was just after the Institute opened. I gave a course as part of a thematic programme similar to the one that's taking place now. After that, I've attended conferences and other schools. I've always maintained a close contact with Javier Parcet and Fernando Lledó. It's great to have an institute like this in Spain that generates a lot of activity and provides so many opportunities. At my university, I've contracted several postdoctoral students who've trained at the ICMAT.

Q: Your research field is in operator algebras. What's the most important feature in this area?

A: The most important for me is that it's connected with many other fields of mathematics. It's origins in the 1930s are linked to mathematical physics, particularly statistical mechanics. Since then, it has progressed a lot and is currently more closely related to group theory and dynamical systems. The fact that it now has so many connections with quantum information theory makes it a very fertile subject.

Q: What's the most significant result of all these years of research?

A: The main result for me is Alain Connes and Uffe Haagerup's <u>classification of injective factors</u>. It's a very complete classification theorem that responds to questions formulated by John von Neumann himself in the 1930s. Alain Connes solved it for all cases except one in 1976, for which he was awarded a Fields Medal. Ten years later, in the mid-1980s, Uffe Haagerup solved it for the remaining case. The question now is to find similar results for nuclear C*-álgebras, and this is the topic of one of the school's courses.

"It's great to have an institute like the ICMAT in Spain; it generates a lot of activity and provides so many opportunities"

Q: What's the importance of von Neumann algebras?

A: I'm interested in the way they come up in group theory and ergodic theory. All the groups are codified in von Neumann algebras in a complex way, and all the group actions and measurable spaces are as well. My research is concerned with understanding which of them are isomorphs and which are not, and developing new tools along the way. In this course, we're speaking about von Neumann algebras that come from free probability theory, for which we're also trying to obtain classification results. For me, it's a very important object because it appears naturally in any field of mathematics where there's some measure. Likewise, the C*-álgebras arise whenever there's any topological structure.



Q: It all boils down to classifying: Why is classification so important for mathematicians?

A: We mathematicians would like to understand the order that appears in these structures. We want to see the full picture. What's more, the interest extends far beyond the final result, and very often it's the methods developed in pursuit of this goal that are enriching for mathematics.

"The language of quantum information theory can use the language of operator algebras"

Q: In the specific case of von Neumann algebras, what are the implications of classification?

A: Von Neumann algebras are connected to another problem posed by Alain Connes, which states that every type II.1 is really contained in the so-called ultra-power of the hyperfinite type II.1 factor. This conjecture seems highly technical, but what it really means is that in a sense all these von Neumann algebras have a kind of finite dimensional approximation. This is a topic that is connected to problems in quantum information theory.

Q: What is the connection between von Neumann algebras and quantum information theory?

A: The language of quantum information theory can use the language of operator algebras. This goes back to von Neumann,

to the way of expressing quantum mechanics, where the observables are operators in a Hilbert space that make up a von Neumann algebra. Many completely new techniques were developed to tackle these classes. Classical probability theory seeks to understand how much information can be contained in different channels of communication. If we are dealing with quantum channels, we have to employ new tools, because we hardly understand the basic principles. The language to express the transmission of information in these channels is the language of operator algebras.

"Quantum groups codify a complex form of symmetry"

Q: You've also worked on quantum groups in the past: What is a quantum group?

A: If we regard the groups as a way of codifying symmetry, the quantum groups codify a more complex form of symmetry. They are algebraic structures more complex than the groups; in fact, the groups are a special case.

Q: What's the motivation of these new structures?

A: They arise naturally in different types of context. For example, they are closely connected to what we call tensor categories, and the language of tensor categories is widely used in the most formal aspects of quantum computers.



Stefaan Vaes gave a course as part of the first school of the 'Operator Algebras, Groups and Applications to Quantum Information' thematic programme

"Entanglement enables us to make teleportation and quantum cryptography a reality"

Interview with Barbara Kraus, researcher at Innsbruck University and speaker at one of the conferences in the "Operator Algebras, Groups and Applications to Quantum Information" thematic programme

Barbara Kraus is head of the Entanglement Theory and Quantum Information Theory group at the University of Innsbruck in Austria. She was one of the speakers at the second conference that was held as part of the "Operator Algebras, Groups and Applications to Quantum Information" thematic programme at the ICMAT. She is a habitual collaborator with Spanish researchers such as Julio de Vicente (who was a postdoc researcher in her group) and David Pérez García. Kraus says that the field of quantum information theory enjoys very good health in Madrid. We spoke to her after her talk on "Entanglement and symmetries of multipartite systems".

Ignacio del Amo and Ágata A. Timón García-Longoria.

Q: What is quantum information theory and how does it differ from classical information theory?

A: Quantum information theory combines classical information theory with quantum physics. Quantum physics provides many possibilities that we don't have in the classical context. I normally use the following metaphor: Classical information is like a switch that is either on or off. It's a bit with a value of 0 or 1. However, in quantum mechanics this light switch can be simultaneously both on and off, not only with a certain probability, but really at the same time. In quantum physics, this is known as the *superposition principle*, and opens up doors to new and interesting possibilities. For instance, in the context of data processing it is possible to perform certain protocols that cannot be done in the classical way or which require more time. That's what quantum information theory is about.

Q: In the quantum context, what is the physical mechanism that allows you to keep information and transfer it?

A: The individual atoms and photons obey this superposition rule. Their behaviour can only be described by quantum physics; doing it with classical physics isn't enough. These phenomena can be used in different ways to create tools, such as quantum computers or quantum cryptographies, which employ these properties more securely and more quickly. You can also use the theory, and the theoretical tools that have been developed, such as the entanglement theory, not only to devise new technologies but also in other fields of physics like condensed matter physics.

Q: What role does mathematics play in all this?

A: It's both the language and the tool. Mathematics is necessary to show that things work, to predict phenomena and to

understand the theory better in order to arrive at fundamental problems. Quantum information theory brings together classical information theory, mathematics and experiments. We don't know if quantum physics is the correct theory. Maybe the same thing could happen as in the case of classical physics; one hundred years ago, it was observed that it wasn't the right theory for describing small systems, and for that it was necessary to develop quantum theory. Now, many experiments exist that verify the predictions made on the basis of this theory. In other words, right now quantum physics has not been invalidated in any experiment; on the contrary, it has been verified in many tests. This experimental approach has to go hand in hand with mathematics, which is how the theory is being developed. Mathematics is essential and without it we couldn't do anything.

"Mathematics is both the language and the tool for describing the quantum world"

Q: Your research is based on mathematics, but also on computer science and physics. What do you find particularly stimulating about this interaction?

A: First of all, you learn an awful lot. I've always found it very appealing how different disciplines focus on the same problem from different angles. If you're stuck with some problem in quantum information or entanglement theory and you talk to a mathematician belonging to a totally different field, you'll see the question from a completely different perspective. It can be very fruitful.

Q: You are particularly involved in the study of entanglement in many-body systems. Could you tell us a little about that?

A: Entanglement is a very strong correlation. We've spoken about a simple system in which superposition may exist: an atom that



can be in an excited state and a ground state at the same time. Now we've got two systems in which this superposition is also found, which means that these systems are strongly correlated. Imagine that we have two dice and we create an entanglement between them. Then I keep one here in Madrid and we send the other one to Barcelona. If I roll my dice, I can get any number between 1 and 6 with the same probability, and the same would happen in Barcelona. However, what happens with entanglement is that if I throw the dice and get a 2, I know that when the dice is thrown in Barcelona you'll also get a 2. This doesn't happen in classical physics. The fact is that these two dice don't really exist, otherwise casinos would be faced with a serious problem. Nevertheless, in quantum physics they do exist, and this enables us to do something new, for example, teleportation (that is, send information from one place to another with no channel in between) or quantum cryptography.

"I've always found it very appealing how different disciplines focus on the same problem"

Q: What problems are you actually working on?

A: One the one hand, we're working on very abstract theories, such as entanglement theory, where we want to understand better what these strong correlations are when we have more than two systems. For two systems we understand them pretty well, but for more than two there are many important open problems in condensed matter physics. We're also looking for new applications for many-body systems. On the other hand, we're also working on quantum computation. We want to find new algorithms that can be executed faster in small quantum computers.

Q: What kinds of problems can only be solved with quantum computers?

A: One example would be teleportation. It's just not possible for it to work in the classical way. Another important application related to this is cryptography, which is something we all use every day to withdraw money from an ATM machine or to communicate via the internet. At the moment, we don't know if the classical protocols are secure vis-a-vis quantum computation. Cryptographic protocols exist that are based on problems that currently take a long time to solve. However, it's not been proven mathematically that they do take a long time. We think that this is the case, but we haven't been able to prove it.

Q: How can you prove that a quantum system is safe?

A: If photons are used to transmit data, you can be sure that if someone tries to tap into this information it will be detected and the communication will be aborted.

Q: What other issues are you interested in at the moment?

A: Another current topic of interest is the verification of processing in quantum computers. There are a lot of groups working on the development of quantum computers, and these computers are already big enough to solve problems much faster than classical computers, including tasks that couldn't even be attempted until now. Even so, how can we verify the results? We've got to find ways of checking quantum computers.

Q: In addition to all that, what do you think are the most important challenges in this field?

A: One of the challenges is to go beyond the classical vision of things. Another would be to solve a series of fundamental

problems in condensed matter physics that are still awaiting an answer. They're very different fields, but you can use tools from quantum information theory to investigate many-body systems. All these topics are related in some way. If you obtain a result in one field, you can use it in another.

Q: Another difficulty to overcome in order to build quantum computers is size. Could you explain a little about that?

A: Yes. You need a quantum system that is big enough to process the information. However, if the objects are very small, for example, atoms, they have to be controlled very precisely. You can't just go on stacking them up until you get the material you want. It has to be done in such a way that the atoms don't interact with other photons that might be flying around. It's necessary to isolate and manipulate the system very carefully, and for that you need very clean systems [that is, it shouldn't contain impurities and should be well insulated against any disruption from outside].

Q: Another issue is quantum decoherence. What's that about?

A: If the atoms interact with other particles in the environment, it causes what is known as decoherence. This means that the atom ceases to be in the superposition state, so the data it contains is no longer very clear, and this error spreads during processing and the result you get is dirty or confusing.

"We have to find ways of verifying quantum computers"

Q: How do you think this will affect quantum computing in the future?

A: There are some very obvious applications, such a simulations that will enable us to learn much more about condensed matter systems. However, we can go much farther, and I imagine we'll see how this works out in the coming years.



Barbara Kraus was speaker at one of the conferences in the "Operator Algebras, Groups and Applications to Quantum Information" thematic programme

SHE DOES MATHS: Isabel Fernández



Isabel Fernández came to the ICMAT to give one of the plenary talks at the $\ensuremath{\mathsf{BYMAT}}$ conference

Research fields:

Differential geometry, geometric analysis, variational problems related to the area, surfaces of constant mean curvature.

Laura Moreno Iraola. Isabel Fernández (University of Seville) always had a predilection for geometry, so when she finished university she applied for a research initiation grant at the University of Granada Geometry Department, where she studied and completed her thesis. Since then she has remained in close collaboration with her colleagues there. Since 2007, she has been a professor at the University of Seville Department of Applied Mathematics I and the Superior Technical School of Computer Engineering, where she combines teaching with research work in the fields of geometric analysis and differential geometry. She specializes in the study of surfaces that minimize area (minimal surfaces or surface of constant mean curvature) or which present some special property relative to their curvature.

Minimal surfaces are those with zero mean curvature. They arise in the search for surfaces that, among all those having the same contour curve, cover the least possible area. From the physical point of view, they correspond to soap bubbles, geometric elements the study of which was introduced by the physicist Joseph Plateau. Fernández has conducted research into these surfaces in the euclidian space R³ and other homogeneous spacesy. Although they fall within the field of geometry, techniques from other areas of mathematics are used to study them, such as partial differential equations, measure theory and algebraic geometry.

Fernández is currently studying more general types of surfaces, the so-called Weingarten surfaces, which are "those in which a relation exists between their mean curve and their Gauss curve", she explains.

Fernández, who is also the secretary of the University of Seville Institute of Mathematics (IMUS) and member of the Board of the Royal Spanish Mathematical Society (*Real Sociedad Matemática Española* - RSME), can also boast of being the first Spanish woman to have been invited to deliver a talk at the International Congress of Mathematicians (ICM), the foremost international event in the field of mathematics. Her paper entitled "Holomorphic quadratic differentials and the Bernstein problem in Heisenberg space", cowritten by Pablo Mira, her friend and collaborator, was published in the journal "Transactions of the American Mathematical Society" in Isabel Fernández (Linares, Jaén, 1979) has been a professor at the University of Seville since 2007, where she combines teaching with research work in the field of differential geometry. Her most important result to date has been her solving of the Bernstein problem in Heisenberg space, together with Pablo Mira, her collaborator and friend. This result meant the complete classification of the minimal surfaces that are graphs of integer functions in that space. It made her the first Spanish woman to be invited to give a talk at the 26th International Congress of Mathematicians (ICM), the foremost mathematical event in the world, held in Hyderabad (India) in 2010. She is currently a member of the Board of the Royal Spanish Mathematical Society (Real Sociedad Matemática Española - RSME) and secretary of the University of Seville Institute of Mathematics (IMUS). In May 2019, Fernández came to the ICMAT to give one of the plenary talks at the BYMAT (Bringing Young Mathematicians Together) conference.

2009 and made her deserving of that distinction. In that paper, the two geometers described the resolution of the so-called *Bernstein problem* in Heisenberg space, in which they completely classified the minimal surfaces that are integer graphs in that space. As Fernández remarks: "We were both invited, which was something completely unexpected, just like the repercussion it was to have later. It was an incredible experience. It was an honour to be invited to give a talk at the most important mathematical congress in the world". Fernández and Mira delivered their talk entitled "Constant mean curvature surfaces in 3-dimensional Thurston geometries" at the ICM held in Hyderabad (India) in August 2010.

Fernández has also participated assiduously for many years in outreach activities. In particular, she frequently gives talks and is currently collaborating in a production of scientific theatre entitled "Women scientists: past, present and future", which has already featured in different outreach events such as Science in Action, where it was awarded the first prize in the category 'Mise en Scene' at the 19th edition held in 2018. This production, which has already been seen by more than 10,000 pupils from 30 different locations, has also received the University of Seville prize at the 2019 Scientific Outreach. Says Fernández: "It was an idea of Francisco Vega, a lab technician colleague at the University of Seville. He has a daughter who was asked by her school to do an assignment on scientists. He then realized that she didn't know anything about women scientists because only men were mentioned in her textbooks, and if women were mentioned they were always the same ones, such as Marie Curie". So it occured to him that she should mention women scientists and try to stimulate scientific vocation in boys and girls by means of theatre, because he thought this would help them to learn more about the story of women in science. She goes on to say that: "He got in touch with me and four other colleagues from the university (Clara Grima, Adela Muñoz, Mª José Rodríguez and Mª Carmen Romero) and we all thought it was a great project. At the beginnng of the show, each one of us played the part of a woman scientist from the past. I took the role of Hypatia and the others of Hedy Lamarr, Rosalind Franklin, Marie Curie and Ada Lovelace. At the end, we all came out on stage to introduce ourselves and tell the audience who we were and what our work was all about. The idea is to present more real and relevant examples so that young people realise that you don't have to be a genius to devote yourself to science".



"I suppose my dedication to mathematics comes from my family"

Luis Martínez (Murcia, 1994) grew up among mathematicians, although he didn't opt for this discipline until he completed his double degree in Mathematics and Physical Engineering at the Universidad Politècnica de Catalunya. After that, he specialized in mathematics with the Master's Program in Mathematics at the University of Bonn (Germany), which subsequently led him to the ICMAT. This September he has begun his thesis on the quasi- geostrophic equation under the supervision of Diego Córdoba (ICMAT-UAM) thanks to a grant from the European Research Council (ERC), which he regards as the chance to set out on a career in research as well as discovering a new city.



Luis Martínez Zoroa

Raquel G. Molina. Luis Martínez has just started work on his thesis at the ICMAT. Under the supervision of Diego Córdoba, scientific director of the Severo Ochoa Project at the centre and beneficiary of a prestigious ERC Advanced Grant, this predoctoral student will devote the next years to research on the quasi-geostrophic equation, which models the flow of air in the lower layers of the atmosphere and has applications to the study of the formation of hot and cold air fronts.

His relation with mathematics began at an early age. The son, grandson and nephew of mathematicians, he showed promise in this field while still very young. "I suppose it comes from my family" says Martínez. He took part in the Spanish Mathematical Olympiad three times, coming sixth on the last occasion, which enabled him to travel to Argentina to take part in the International Olympiad. However, he didn't opt for this discipline until he completed his double degree in Mathematics and Physical Engineering at the Universidad Politècnica de Catalunya (UPC – Technical University of Catalonia). "I wasn't sure whether I wanted to devote myself to mathematics or physics," he says, "so I decided on that degree so I could study both of them".

On completion of his degree he moved to Bonn (Germany), where he specialized in mathematics with the Master's program in Mathematics, about which he says: "In comparison with higher education in Spain, there was a wider variety of subjects there, which can also be a drawback because you specialize less". Now he's excited about the chance to experience a change of scenery. "It's the first time I've been to Madrid; I've not even visited the Retiro Park before".

"Research has always seemed an enjoyable option because it's a challenge," says Martínez, who is optimistic about starting more "real" research work than he has done before. Until now he knew almost nothing about fluid dynamics, the field to which his PhD belongs. It was his mother who recommended the project to him. "It seemed very interesting to me, and my thesis supervisor is a fantastic researcher, so I decided to make the most of this great opportunity". He has great expectations of his training experience at the ICMAT. "It's very gratifying to be able to spend some time trying to achieve something and making it work out well".

How does he see the future? "Apart from being engaged on a project I enjoy, I have no other long-term expectations". He's aware that research is a career in which it's difficult to find stability or tranquillity, but he loses no sleep over it at the moment. "I don't know where I'll be in six years' time", he says, "but I'd like to find a permanent position here in Spain". This would suit him because he'd like to keep in close contact with his family and friends. The one thing that is certain at the moment is that he'll continue working on his thesis until August 2023.

"Optimal transport is a powerful tool in mathematical finance"

Interview with Jan Maas, researcher at the Institute of Science and Technology in Austria and invited speaker at the second BYMAT conference

Jan Maas is an international expert in stochastic partial differential equations, optimal transport and stochastic analysis. He currently works at the Institute of Science and Technology in Austria and recently visited Madrid to give a plenary talk at the second "Bringing Young Mathematicians Together" (<u>BYMAT</u>) conference. We got together outside the conference building on a sunny day to chat a little.



Ignacio del Amo

Q: What is a stochastic process?

A: It's a mathematical concept that describes any phenomenon that is subject to randomness. It very often depends on time and space, but there's always some random component. This could be due to the intrinsic stochastic nature of the problem, the uncertainty of the observer, or the noise or perturbation present in the system.

Q: Could you give us some real-life examples of that?

A: These phenomena are found everywhere. One example would be the turbulent motion around an aircraft wing. Another classical example is in finance; interest rates, for instance, which are modelled using Brownian motion or stochastic differential equations. Yet another is found in population dynamics, and basically everywhere there is noise; in meteorology, weather forecasting and so on.

"Stochastic phenomena are found everywhere"

Q: How are these processes studied mathematically?

A: Very often they are described using stochastic differential equations. The theory emerged in the 1940s thanks to the work of Kiyoshi Itô. A lot of developments have been made recently; for example, the theory of rough paths by Terry Lyons or the theory of stochastic partial differential equations. The theory of regularity structures, developed by Martin Hairer (for which he received a Fields Medal), is also very recent and is able to deal with random signals in time and space. This is a field in which an

Jan Maas was speaker at the second BYMAT conference



interaction exists between probability theory and other areas of mathematics, and sometimes there are also ideas coming from geometry and analysis.

"Optimal transport is a tool for understanding other problems"

Q: You also do research in optimal transport. What kind of problems do you tackle there?

A: The classical problem there is, given an initial distribution of some material: How can it be transported so that it goes from certain initial points to certain final points, in such a way that it minimizes the total cost of transport? This is an old problem posed by Gaspard Monge that goes back to the 18th century. More recently, in the 1940s, Leonid Kantorovich formulated the problem in modern language. These days, it's not so much about studying transport directly, but rather using optimal transport as a tool for understanding other problems. In a sense, optimal transport provides us with a means of defining distances between probability measures, so that it adapts very well to the geometry on the underlying space and enables the geometry of the given space to emerge as probability measures on that space. This is useful for both applied problems in statistics and for describing evolution equations such as partial differential equations. Optimal transport can also be used for obtaining more information about the underlying space.

"In data science and machine learning there's a great deal of excitement about the ideas regarding optimal transport"

Q: And, of course, this has applications for industry...

A: Yes, absolutely; in applications to planning problems (obvious by definition) and to economics. Over recent years, it has become quite a powerful tool in mathematical finance. Furthermore, in data science and machine learning there's a great deal of excitement about the ideas regarding optimal transport.

Q: What are the main challenges and the open problems in these areas?

A: One of the main challenges in stochastic analysis and stochastic differential equations concerns questions related to universality; that is, moving from a microscopic description of a random system to a macroscopic description, as well as showing that the latter isn't sensitive to the details of the underlying models. Where optimal transport is concerned, I think it's difficult to point to a single big open problem, but personally I'm interested in the applications to problems arising from quantum information theory. In this case, we're not looking at probability distributions, but rather at density matrices. This is a promising line to pursue, and it would be interesting to find applications in this area.

"I'm interested in the applications of optimal transport to problems arising from quantum information theory"

Q: In addition to that, what other problems are you working on at the moment?

A: I'm trying to go beyond the limits of continuous to discrete optimal transport. Many of the examples concern what ought to happen, but sometimes the limit of discrete optimal transport is determined by something more complicated. Generally speaking, if you work on discrete mathematics you're dealing with objects that are easier to work with, in the sense that it takes less effort to define them rigorously, but they're more difficult to deal with than in the continuous case, because many formulas that are true within the continuous limit do not hold at the discrete level. What's more, at the continuous level, there are often several equivalent ways of addressing a problem. For example, in optimal transport there's a static problem and a dynamic problem, and both are the same in the continuous setting, but not in the discrete world. So you don't have as many tools and it's more difficult to tackle the problem.

Translation and editing: Ágata Timón García-Longoria



The second edition of the BYMAT conference was celebrated between May, 20 and 24, 2019

"The main difficulty in the mathematics classroom is the existence of a great diversity"

Interview: Rafael Ramírez Uclés, researcher in the didactics of mathematics and professor at the University of Granada

Rafael Ramírez Uclés belongs to the Department of the Didactics of Mathematics at the University of Granada, where he teaches degree classes in Primary Education and master classes in Secondary Education and Baccalaureate as well as Vocational Training, Language Teaching and the Didactics of Mathematics. At the same university, he is also a researcher in the teaching and learning of geometry: spatial sense and mathematical talent. Ramírez gave the plenary lecture "Enrich the mathematical talent" at the BYMAT young researchers conference, held at the ICMAT in May 2019.

Laura Moreno Iraola

Q: What is your research line the didactics of mathematics about?

A: We work in different areas. In particular, I've specialized in research concerning students who are highly gifted in mathematics and how to meet their educational needs. Above all, we focus on visualization skills, and as I explained in an <u>article</u> published in the ICMAT section 'Coffee and Theorems' in the Spanish daily El País, these skills consist of the development and proficiency of spatial sense in order to understand the world that surrounds us. Applied to mathematics, this approach concerns the teaching of geometry beyond just formulas and the memorization of terms in order to appreciate the geometry that surrounds us. It's also possible to understand mathematical concepts in a simpler way through visualization, by means of which we connect our perception of the world with the equations that explain it.

Furthermore, together with a group from the University of Valencia, we're studying how these mathematically gifted students understand proofs. Finally, we also study the *functional thinking* of highly gifted primary school pupils, as a means of introducing algebra.

Q: Can your conclusions be applied effectively in the classroom?

A: Of course; our aim is to transfer all the research material to teaching processes and see what results we come up with.

For example, we teach our teacher-training and Secondary School and Baccalaureate master students techniques to provide them with skills and resources they can use in the future when it is their turn to teach highly gifted students, as well as the ability to design tasks for the development of spatial sense.

"Mathematics isn't a subject disliked by pupils, at least at a primary level."

Q: Do you respond to teachers' requirements? What would you say are the main problems teachers face when teaching mathematics?

A: The main problem I find is the great diversity that exists in the classroom in Spain. In a class of 25 students you can find boys and girls who lag behind in the curriculum and others who are highly gifted, and they have different levels of learning. In my experience, this doesn't happen in other countries. For instance, last year I was in Boston (USA) visiting different schools, and I discovered that there were often two teachers in each classroom, and each class also had half the number of students we have here. This is a strategy we could adopt in Spain, where teachers need more resources and training in order to deal with this diversity.



Q: What about the methodology? Do you think mathematics can be taught differently?

A: In my opinion, it wouldn't be necessary if teachers were provided with more resources, materials and training in order to tackle the diversity I spoke about earlier. Teachers are well trained and I don't think mathematics is a subject disliked by pupils, at least at a primary level. The difficulty begins in secondary school, because the problems and topics are hard and maybe we're not used to the fact that some things require effort.

"The mathematics syllabus is well organized and provides the appropriate methodological guidelines."

Q: So do you think that the maths syllabus is properly adapted?

A: Yes, if you look at it in detail you can see that it is well organized and provides the appropriate methodological guidelines.

Q: Some people bemoan the enormous difference between the maths taught in secondary and high school and that in degree courses and research. Why does this gap exist?

A: Because the content of the curriculum is progressively changing, and in secondary education, for example, it's not

considered necessary for pupils to do proofs or use a formal language, because of the stage they're at. It's more important for them at secondary level to learn how to solve problems and understand that mathematics is functional. At each stage we must cover what pupils have to learn, teach the concepts well so they understand the meaning of what they're studying as well as its uses and applications, so when they move on to higher levels they can deal with more specific and formal aspects of mathematics.

Q: To what extent do you think outreach is enriching for researchers as well as contact with other branches of mathematics, such as non-university teaching?

Outreach is extremely important. There's no doubt that researchers have to write papers for publication in prestigious scientific journals to find an outlet for their research work and share it with other members of their community. On the other hand, outreach enables your work to reach a wider public. It can make society more interested in science - in our case, mathematics - and thus help society evolve progressively. The efforts made by many researchers to make the highly complicated concepts they work on more understandable for the general public are worthwhile, and something that should be greatly appreciated.



Rafael Ramírez gave the plenary lecture "Enrich the mathematical talent" at the BYMAT conference

ICMAT QUESTIONNAIRE

Getting to know some of the participants of the poster session held at the 2019 BYMAT conference

The second edition of the BYMAT young researchers conference, organized in May 2019 by the ICMAT, included some additions to the programme that did not feature in the previous event. One of these was a <u>poster session</u> that provided its participants with the chance to share their research work, most of which formed part of their PhD theses, with the other attendees belonging to institutions from all over the world. Some of the young researchers who presented their preliminary steps in research to their colleagues were: Hamzah Bakhti (Mohammed V University of Rabat, Morocco), María de Gádor Cabrera Padilla (University of Almería), Emily Quintero (University of Alcalá) and Jeyms Villanueva-Cantillo (University of Cádiz).

Ignacio del Amo and Laura Moreno Iraola

<u>Hamzah Bakhti</u>

Institution: Mohammed V University of Rabat (Morocco) and Kiel University of Applied Sciences (Germany).

Position: PhD student in applied mathematics.

Thesis director: Lahcen Azrar (Mohammet V University of Rabat, Morocco)

Supervisor: Mohammed Es-Souni (Institute for Materials and Surface Technology, IMST, and Kiel University of Applied Sciences, Germany).

What is the subject of your doctoral thesis? I'm working in different fields of applied mathematics, such as materials science and biology. Specifically, together with my collaborators I'm trying to devise mathematical models for describing different phenomena and for conducting simulations to understand them. On my poster, I explain how we apply mathematics to the study of cardiovascular diseases.

I'm currently on a stay at the University of Applied Sciences in Kiel, Germany, where we're carrying out experimental research in which we simulate different real problems, and then check and compare them with the numerical results.

Which direction would you like your professional career to take?

I'm not sure at the moment, because the only direction open to me in Morocco is as a university lecturer, while in Spain you can combine teaching with research and there are many companies that employ mathematicians.

What has your experience at the BYMAT young researchers conference been like?

A very enriching experience; I also attended the first edition and was pleasantly surprised at the atmosphere there, but this second event has been even better. I think it's a very good



Hamzah Bakhti

initiative, because it brings together young mathematicians from all over the world. Last year I made a lot of friends whom I've met again this year. What's more, the atmosphere on campus is excellent and I love Madrid. I really hope that BYMAT goes on organizing conferences and other activities in which I can take part.

María de Gádor Cabrera Padilla

Institution: University of Almería.

Position: PhD student in functional analysis.

Thesis director: Antonio Jiménez Vargas (University of Almería).

Supervisors: Moisés Villegas (University of Cádiz) and Javier Alejandro Chávez (University of Oklahoma, USA).

What is the subject of your doctoral thesis? I study duality theory for ideals of Lipschitz operators. In particular, I'm working on the doctoral research of the mathematician Alexandre Grothendieck and on his *Résumé de Sao Paulo*, in



María de Gádor Cabrera Padilla

which the topological dual of the tensor product between Banach spaces is related to linear and continuous operator spaces. We've adapted this theory to Lipschitz functions that present ideal properties different from those given for linear and continuous operators.

What course would you like your professional career to take?

I'd like to continue working in research as a postdoctoral scientist and, if possible, combine this with university teaching.

What has your experience at this BYMAT young researchers conference been like?

I was at this conference in 2018 and returned again this second edition, because I love meeting other young researchers and speaking with them about their own research work. This is something enriching for all our work, even though it might concern other areas. I've met people who are studying problems similar to mine; we've developed a good relationship and intend to remain in contact.



Emily Quintero

Emily Quintero

Institution: University of Alcalá (UAH).

Position: Teacher in Venezuela and PhD student in the field of numerical and symbolic algorithms, and applications to curves and surfaces.

Thesis director: Juan Gerardo Alcázar Arribas (University of Alcalá).

What's the subject of your doctoral thesis?

My research work consists of finding affine equivalences between two algebraic varieties. We're currently focusing on the equivalences between two rational ruled surfaces. The idea is to create an algorithm that tells us if two given surfaces are equivalent and is also able to calculate them. We started by investigating symmetries of a ruled surface, and throughout 2019 we've been able to generalize the work for general affine equivalences between any two ruled surfaces. The entry algorithm consists of two ruled surfaces and the exit consists the equivalences affine to the surfaces. What's interesting about this is that it's an R³ problem, but with the method we use we lower the dimension in order to find



The poster session was one of the additions to the programme that did not feature in the previous BYMAT edition



Participants had the chance to share their research work with the other attendees

birational functions from R^2 to R^2 , and by calculating them we'll arrive at the affine equivalences between those two surfaces.

Which direction would you like your professional career to take?

At the moment I'm working as teacher in Venezuela and I've obtained a grant from the Carolina Foundation to do my PhD in Spain, so I'd like to continue my research work and, if possible, combine it with teaching.

What has your experience at this BYMAT young researchers conference been like?

I think it's a very good opportunity for we students to present our work, and at the same time to find out what our colleagues are doing. I was here in the first edition to give a talk and in 2019 I've participated in the poster session. There's a very nice atmosphere here and I've been pleasantly surprised by the quality of the work.

Jeyms Villanueva-Cantillo

Institution: University of Cádiz and Simón Bolívar University (Colombia).

Position: PhD student in the field of statistics and operations research in Cádiz, and university lecturer and secondary school teacher in Colombia.

Thesis director: Manuel Muñoz Márquez (University of Cádiz).

What's the subject of your doctoral thesis? The calculation of critical values for the selection of variables and data envelopment analysis (DEA), a technique introduced about 40 years ago, but still without any criterion for defining what variables to include in the model. For example, in regression analysis, we have p values for the variables, so we can decide which variables to include and which not, but for this technique those critical values don't exist.

In which direction would you like your professional career to lead?

In recent years I've been teaching, but after my PhD I'd like to concentrate more on research while still teaching at university.

What has your experience at this BYMAT young researchers conference been like?

It's been an excellent experience. I've learned a lot in the statistics sessions; for example, about questions concerning neural networks and codification, etc. In general, the conference has enabled me to acquire a more overall view of mathematics. In Colombia, the sciences, and especially mathematics, are not nearly as strong as they are here. Meeting so many different people who are working in so many diverse fields of mathematics is what I've enjoyed the most. This is my first visit here and it's been very surprising.



Jeyms Villanueva-Cantillo



SCIENTIFIC REVIEW: Singular integrals in quantum euclidean spaces

Original title: "Singular integrals in quantum euclidean spaces".

Authors: Adrián M. González-Pérez (KU Leuven, Bélgica), Marius Junge (University of Illinois at Urbana, EE.UU.) and Javier Parcet (ICMAT) Source: Memoirs of the American Mathematical Society

Date of publication: accepted, to be published

One of the most dramatic shifts in our understanding of the physical world occurred in 1925, when Heisenberg proved that quantum phenomena could be deduced from the equations of Newtonian physics if we interpreted the variables that depend on time as infinite matrices. John von Neumann proposed to model Heisenberg matrices as adjoint operators over Hilbert spaces. The theory of von Neumann algebras is a noncommutative or quantum form of measure theory and provides a rigorous mathematical framework for matrix mechanics. The von Neumann Programme responds to the need of "quantizing mathematics" in order to complete the classical/relativistic notions of measure and geometry. It constitutes a challenge of extraordinary magnitude that has exceeded the contributions of von Neumann himself. Indeed, today we are able to speak of noncommutative geometry, quantum probability, operator spaces, quantum groups and so on. Apart from its importance for mathematics, connections exist with theoretical physics; in string theory, quantum field theory and quantum information. All of this explains the necessity to develop the von Neumann Programme in other directions, one of which is harmonic analysis.

In 1980, Alain Connes introduced noncommutative geometry [5] as an extension of differential geometry over "noncommutative manifolds" in the operator algebra language introduced by von Neumann. The archetypes of noncommutative manifolds are quantum tori and quantum Euclidean spaces. Given Θ a real antisymmetric matrix n x n, the associated n-dimensional torus is defined (vaguely) as the algebra generated by unitary operators u_{η} , u_{η} , \dots , u_{s} that satisfy the canonical commutation relations $u_i u_k = \exp(2\pi i \Theta_{ik}) u_k u_i$. When $\Theta=0$, the u_i are the primary characters $\dot{x} \rightarrow \exp(2\pi i \dot{x})$ and the associated algebra that of the functions (smooth, continuous, bounded...) over the classical torus $\mathbf{R}/\mathbf{Z} \times ... \times \mathbf{R}/\mathbf{Z}$. When Θ does not cancel, the algebra is noncommutative and is known as a quantum torus. The quantum Euclidean spaces admit a similar definition; the Heisenberg-Weyl algebra is the most well-known model. Connes introduced in [4] pseudo-differential operators in quantum tori with the aim of extending the Atiyah-Singer index theorem in this context. The insight provided by Connes' approach was subsequently confirmed by the Gauss-Bonnet theorem for quantum tori [6, 7] and a solid elliptic operator theory. In its 40 years of history, noncommutative geometry has also developed connections with number theory and string theory [12].

Pseudo-differential operator theory emerged in the mid-1960s with the work conducted by Kohn, Nirenberg and Hörmander [9]. The idea is to use the Fourier transform to represent differential operators $L = \sum_{l \neq l \le m} \sigma_{\alpha}(x) \partial_x^{\alpha}$ that can be inverted except for an error term. Pseudo-differential operators can be interpreted as singular integrals. Calderón-Zygmund theory [3] --paradigm of modern harmonic analysis-- then provides *p*-Sobolev-type estimates of the approximation and error terms, which leads to the deepest results of the theory. Unfortunately, the work by Connes and his collaborators does not include this class of estimates due to deep obstructions to develop the theory of singular integrals over noncommutative L_{α} spaces, defined over von Neumann algebras.

In the work reported here, the core of singular integral theory and pseudo-differential calculus are established on the model algebras for noncommutative geometry: quantum forms of tori and Euclidean spaces. The latter –also known as Moyal deformations in theoretical physics or CCR algebras in quantum probability- include the Heisenberg-Weyl algebra determined by the position and the momentum in quantum mechanics. These results on pseudo-differential operators go beyond the work of Connes, thanks to a new form of the Calderón-Zygmund theory in these algebras, which are developed in the same work and which crucially includes general kerrnels which are not of convolution type. This enables to deduce L_p boundedness and p-Sobolev estimates for regular, exotic and forbidden symbols in the expected ranges. In L_2 the authors also generalize the Bourdaud and Calderón-Vaillancourt theorems [1, 2] for exotic and forbidden symbols. All the foregoing establishes the quantum forms of the most famous results of pseudo-differential operator theory [13]. As an application of these methods, L_p regularity of solutions to the first elliptic PDEs in von Neumann algebras are proved.

Finally, it is worth pointing out that noncommutative Calderón-Zygmund theory has precedents in the work of the authors with interesting connections in geometric group theory and Lipschitz operator functions. However, unlike in the previous results [8, 10], this is the first model that works in *purely noncommutative* algebras; that is, algebras that contain no copies of doubling metric spaces in the form of tensor products or crossed products. Recently, some of the authors have developed in [11] an *algebraic form of Calderón-Zygmund theory* that is valid in general von Neumann algebras equipped with a Markov process that satisfies strictly algebraic conditions.

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SCIENTIFIC REVIEW: On the solvability of the Dirichlet problem for elliptic operators

Título original: "Perturbations of elliptic operators in 1-sided chord-arc domains. Part I: Small and large perturbation for symmetric operators".

Autores: Juan Cavero (ICMAT), Steve Hofmann (Universidad de Missouri, EE UU) y José María Martell (ICMAT).

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Partial differential equations (PDEs) enable the modelling of phenomena arising from natural, social and economic sciences. One example is provided by the equations that define the evolution of diffusion (such as the heating of a solid body to which a heat source is applied). In this type of problems, one seeks for solutions to an equation associated with a differential operator and defined in a certain region, prescribing the values taken by the solution at the boundary of that region. Finding the solution to a differential equation, when the boundary value is known, is what is called solving the *Dirichlet problem* associated to the differential operator under consideration.

The Dirichlet problem was originally posed for the *Laplace* equation – paradigm of the second-order elliptic partial differential equations – and it can be solved nowadays for many differential operators. The class of data that we consider in the boundary is crucial for solving the Dirichlet problem. For the Laplace equation, with continuous boundary data and with solutions that are continuous up to the boundary of the region, the well-known *Wiener criterion* characterizes the class of regions in which the Dirichlet problem can be solved.

Apart from the Laplace operator, however, it is interesting to study the same problem for more general operators, such as the second order *divergence* form elliptic operators, where the coefficients may vary according to the position. Furthermore, the situation becomes more complicated when studying problems in which the boundary values show worse behaviours. For example, we can pose the *L*^{*p*}-Dirichlet problem (with 1) where theboundary data belong to L^p , the space of functions whose pthpower is integrable, and in which the solutions coincide with their boundary values, in the sense of approximation to the boundary in regions that are generalized cones (that is, *non-tangential* regions). Thus, in this formulation, boundary values that are not necessarily continuous are allowed, and may even possess singularities. In such cases, is it possible to establish what conditions are required in order to ensure that the L^p-Dirichlet problem is solvable in the previous sense?

Different methods exist for approaching this question. In the article "Perturbations of elliptic operators in 1-sided chord-arc domains", Juan Cavero (ICMAT), Steve Hofmann (University of Missouri, USA) and José María Martell (ICMAT) start from a given operator for which the solvability of the L^p -Dirichlet problem is known (with a certain p, 1), and study to what extent it is possible to modify the operator in question so that one is able to solve a similar Dirichlet problem for the new operator. In other words, it involves determining which*perturbations*of the original operator still admit solutions to the Dirichlet problem.

In their article, Cavero, Hofmann and Martell work in rough domains that are *open and path-connected* and whose boundary has *codimension* 1. Assuming that, in a quantitative or scaleinvariant sense, they determine certain conditions that the operators should satisfy in order to ensure the *solvability* desired. Specifically, the conditions are about the difference in the coefficients of the original and the perturbed operator, where both are real symmetric. In the case where the discrepancy between the coefficients is sufficiently small (in the sense of a certain *Carleson measure*), the authors have proved that solving the *L*^p-Dirichlet problem for the original operator translates into solving the same problem (with the same value *p*) for the perturbed operator. More generally, for large discrepancies between the coefficients, what they show is that solving the *L*^p-Dirichlet problem for the original operator enables the solvability of the *L*^q-Dirichlet problem for the original operator translates into solving the *L*^q-Dirichlet problem for the original operator enables the solvability of the *L*^q-Dirichlet problem for the original operator for another value $q \in (1, \infty)$ that is not necessarily *p*.

These results extend the work of Fefferman-Kenig-Pipher in [3] and Milakis-Pipher-Toro in [5, 6], who considered domains with a certain degree of regularity. Now, Cavero, Hofmann and Martell employ an alternative method that enables more general domains to be taken into account.

In the work in question, some applications are provided in which it is possible to establish geometric consequences from the solvability of the L^p -Dirichlet problem. For example, if a symmetric perturbation of an operator with coefficients having certain regularity, such as the *Laplacian*, possesses the property that the L^p -Dirichlet problem is solvable for some $p \in \{1, \infty\}$, then the perturbation theory establishes that it is possible to solve the L^q -Dirichlet problem for some other $q \in \{1, \infty\}$. This, together with the work by J. Azzam, S. Hofmann, J. M. Martell, K. Nyström and T. Toro [2], shows that the domain must have an open exterior in a quantitative sense, and therefore its boundary has a certain degree of regularity (it is *uniformly rectifiable*). As one may see, geometric information is thereby obtained about the region in which it is possible to solve a Dirichlet problem.

The techniques used in the foregoing questions require the inclusion as a hypothesis that the region should satisfy a certain notion of strong connectivity. Without this hypothesis, it is not known if the previous perturbation results are valid, which throws up an interesting challenge in the field. J. Azzam, S. Hofmann, J. M. Martell, M. Mourgoglou and X. Tolsa have recently presented a characterization of the regions in which it is possible to solve the Dirichlet problem- L^p for some $p \in (1,\infty)$ for the Laplace operator. This geometric characterization requires the boundary to be regular; that is, it should be *uniformly rectifiable* (as may be seen in [4]) as well as a condition of access to portions of the boundary by means of non-tangential paths. In this article [1], which is still in the process of peer-review, the proposed characterization is optimal, since none of the imposed conditions can be eliminated.



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TELL ME ABOUT YOUR THESIS: Bruno Vergara

Thesis title: "Weighted inequalities in Fluid Mechanics and General Relativity: Carleman estimates and cusped traveling waves". Author: Bruno Vergara (ICMAT-UAM) Director: Alberto Enciso (ICMAT) Date of submission: July 16th, 2019

Raquel G. Molina. Bruno Vergara (February 18th, 1991, Valparaíso, Chile) says that the four years he has spent completing his doctoral thesis on partial differential equations (PDEs) have been extremely satisfying, especially because it has enabled him to collaborate with first-rank researchers. It is thanks to them, and particularly to his thesis supervisor, Alberto Enciso (ICMAT), that in this young researcher's own words: "I've been able to learn what research activity is all about, above and beyond mathematical techniques in themselves". He defines research work as a process full of difficult moments, of setbacks and ups and downs, but which nevertheless proceeds onwards.

His thesis, entitled "Weighted inequalities in Fluid Mechanics and General Relativity: Carleman estimates and cusped traveling waves", is divided into two parts in which singular solutions to partial differential equations are analyzed in two different contexts: General relativity and Fluid Mechanics. In the first part, he seeks to generalize a known result in General Relativity – classical Morawetz estimates – to the case of waves with critically singular potentials and thereby to build new Carleman inequalities. The second part is focused on fluid mechanics equations; specifically, the Whitham equation for waves on the surface of a fluid. Together with his thesis supervisor, Vergara has managed to prove the convexity of singular solutions to this equation, for which no partial results previously existed.

Vergara states that "each problem has required the combination of both known and new ideas". For example, he has used estimates with singular weights and *ad-hoc* growth in order to control a critically singular potential. Another new idea consists of a strategy for finding singular solutions to PEDs with low regularity. To that end, he resorted to the construction of an approximate solution by using a combination of fine asymptotic analysis close to the singularities and computer-assisted estimates.

At present, this mathematician is at the University of Zürich (Switzerland), where he holds a position of postdoctoral researcher and says that in the future he would like to continue in the academic world.

The influence of partial differential equations (PDEs) is such that one is able to say there is no branch of science in which they are not used: Maxwell's equations constitute the cornerstone of electromagnetic theory; the Navier-Stokes equations provide the foundation for hydrodynamics; the quantum revolution in physics is based on the Schrödinger equation, and the Black-Scholes equation was the basis for the financial calculations that during the first decade of this century led to the economic crisis. The success of partial differential equations, with applications also to different branches of theoretical mathematics, resides in their capacity to model an enormous diversity of physical, biological, chemical phenomena, as well as in engineering and economics, among other fields.

A fundamental question in the theory is to determine if a given system of partial differential equations has solutions with *singularities*; that is, points with a different behaviour than the rest, where things do not proceed "smoothly". Likewise, in the models in which the *ambient geometry* presents some type of singularity, it is interesting to understand the effect this may have on the solutions. In my thesis, the singularities appear in two different flavours. On the one hand, through geometric wave equations with *singular potentials*, arising from *boundary problems* in certain types of spacetime belonging to the theory of general relativity. On the other hand, by the study of waves in fluid mechanics, with the profile analysis of a periodic *travelling wave* of maximum height, which exhibits cusp-shaped singularities that are asymptotically of the form $V|_{x|}$.

The main characteristic of the first type of PDEs mentioned above is that they can be written as a regular part plus a *strongly singular* potential which depends on the distance to the boundary of an (n+1)-dimensional Lorentzian manifold. These potentials, which scale like the *Laplace-Beltrami* operator, are called *critically singular* and they are generally difficult to address with standard methods, which are only suitable to obtain estimates in $L_{loc}^{(n+1)/2}$. Wave equations with critical singularities appear naturally in the context of *anti-de Sitter* (AdS)-type spacetimes in theoretical physics; in particular, in the study of *Cauchy problems* with data in the *conformal infinite*. In the simplified model of a cylindrical domain in *Minkowski space*, this leads to the consideration of a potential that diverges as the inverse square of the distance to the boundary of the cylinder and whose intensity plays a role similar to the mass of the fields modelled by the PDE in AdS.

In order to prove uniqueness properties in these geometric wave equations, it is necessary to develop new *Carleman-type* estimates with optimal weights adapted to the geometry of the singularity at the boundary. In general terms, *observability* is tantamount to quantitative uniqueness; that is, it defines a notion of *unique continuation* in which the prescribed Cauchy data at the boundary control a significant energy of the solution. The main result of this part tells us that it suffices to measure the normal derivative at the boundary (the *Neumann data*) for a sufficiently long time interval, which depends solely on the ambient geometry and the intensity of the potential, in order to obtain information about the energy (with weights) of the solutions.

Closely related to the foregoing, an interesting question consists in the construction of a Lorentzian analogue of the *fractional laplacian*. The relation between *covariant* operators in Einsteintype conformally compact manifolds and the *fractional* powers of the *laplacian* has aroused great interest in both *Riemannian* geometry and *nonlocal* elliptic PDEs. One of the results of my doctoral thesis is the extension of this relation to Lorentzian manifolds, thereby proving that the fractional powers of the *standard* wave operator (in flat space) can be constructed as *Dirichlet-Neumann operators* associated with wave equations in the *AdS semi-space*. These asymptotically AdS spacetimes play a vital role in cosmology due to their connection with the AdS/CFT conjecture in string theory, so that this construction is not only interesting from the mathematical point of view, but also shares a certain connection with some questions in theoretical physics.

In the second part of the thesis we study the Whitham equation, which serves as a unidimensional model for the study of surface waves in water as part of fluid mechanics. This nonlocal dispersive equation model admits solutions in the form of solitary waves as well as singular solutions. In the latter case, the equation was popularized as the result of a conjecture regarding the existence of travelling waves of extreme form with cusps that behave like $\sqrt{|x|}$. By means of a delicate global bifurcation argument, in 2016 Ehrnström and Wahlén proved that the Whitham equation presented solutions in the form of greatest height travelling waves with cusps of regularity $C^{1/2}$. This singular behaviour occurs as a result of the balance between two things: the linear term given by a weakly dispersive inhomogeneous operator governing the equation and the non-linearity, which in this case is quadratic, as in the Euler equation from which water waves equation are derived.



Cusped periodic travelling wave solution to the Whitham equation

The convexity of the Whitham greatest height solutions is at the heart of the second part of my thesis. In way similar to the well-known case of the Stokes water waves for the Euler equation, it had been conjectured that the extreme waves of the Whitham model featured a convex profile between consecutive peaks. In the Stoke's case, the convexity property was proved in 2004, almost two decades after showing the existence of waves with corners of 120°. In the Whitham equation, the form of the nonlocal linear operator makes the proof of convexity to follow a completely different strategy to that of Stoke's conjecture. This proof is based on a fixed-point method that combines fine asymptotic analysis close to the cusps and computer-assisted estimates for the construction of approximate solutions.

All these results described in the thesis pave the way to take our research into new directions. Is it possible to make statements about the observability of the wave equations in asymptotically AdS-type spaces? What is the optimal regularity? What would occur if the cylindrical model was changed for another convex domain? In the second part of the thesis dealing with fluid dynamics, the flexibility of the technique employed in the proof of convexity has also enabled us to analyze the issue of global uniqueness in the Whitham model. It would be interesting then to study other models with low regularity and prove the existence of solutions to PDEs with cusps or corners. Many of the road to extending our knowledge about singular solutions to PDEs.



MATHEMATICS TODAY: ICMAT News

Results

Alberto Enciso (ICMAT) awarded a Consolidator Grant from the European Research Council

"Analysis of geometry-driven phenomena in fluid mechanics, PDE's and spectral theory", the project of Alberto Enciso, scientist of the CSIC at the Institute of Mathematical Sciences (ICMAT), has received one of the 301 Consolidator Grants that the European Council of Research (ERC) has given in the last call. It is the only Consolidator granted on this occasion to a Spanish researcher in the field of mathematics.

For five years and with a budget of 1.8 million euros, the researcher will study new properties of partial derivative equations (PDEs) and their application to physics. "To unravel different natural phenomena, we must resort to innovative and increasingly powerful ideas that allow us to understand and describe them," says Enciso, who obtained a Starting Grant from the ERC in 2015.

The project focuses on three areas. First, on the formation of singularities in fluid mechanics, where the goal is to understand how a fluid develops points of explosive or pathological behavior, as occurs when a wave breaks. On the other hand, the project will study the geometric properties of PDEs, not only of fluid mechanics, with emphasis on fluids in equilibrium, but also on quantum mechanics. In this sense, the researchers will try to understand the evolution of *Bose-Einstein condensates*, also called superfluids. Finally, the third area is spectral theory, which describes mathematically the fundamental frequencies of vibration of objects and explains, for example, the sound of a drum.

ICMAT researchers put an end to a Chern conjecture

The so-called *Chern conjecture* has been one of the most important open problems in the area of symplectic topology for more than 20 years. In 2015 it was solved, for spaces of dimension 5, by Francisco Presas, scientist of the CSIC at ICMAT, Roger Casals, then doctoral student of Presas and now member of the University of Davis in California, and Dishant M. Pancholi, from Chennai Mathematical Institute (India). Just a couple of months after Annals of Mathematics –one of the most prestigious scientific journals in the field of mathematics– accepted its paper, another group of mathematicians at Stanford University announced a demonstration of the general case.

In their work, the Stanford group was able to generalize the so-called overtwisted disk -well known in dimension 3- to solve the problem. The 2015 article on Dams was based on another controller model, introduced by mathematician Klaus Niederkruguer in 2005, the *plastikstufe*. In addition to this, there were up to seven alternative definitions of universal controllers, introduced in the previous 15 years. However, the relationship between these notions and the general one proposed by Borman, Eliashberg and Murphy, was unknown, until a few months. "After two years of work, we have managed to demonstrate that all alternative definitions of universal controller are equivalent to the definition introduced by Stanford researchers," says Presas. The article, published in 2019 in the Journal of the American Mathematical Society, has brought an end to the resolution of the Chern conjecture, and, in addition, has validated the work of many mathematicians in the area, done in the last 20 years.



Alberto Enciso has been awarded with a second ERC Grant in 2019



Francisco Presas, one of the authors of the paper about the Chern conjecture, celebrated his 45 anniversary with a congress at the ICMAT in December 2019

Algorithms that coordinate a squadron of drones

These days, drones are used in a variety of situations: in rescue missions after a disaster, in surveillance work over oceans (for example, for tracking the dispersion of plastic material), for the transportation of heavy loads or for recording sporting events. Many of these operations cannot be undertaken with a single aircraft, so a whole fleet is required to operate in a coordinated manner. A project known as "Descentralized strategies for cooperative robotic swarms", designed by ICMAT researcher Leonardo Colombo, has been awarded a grant of 300,000 euros by "la Caixa" Foundation for a 3-year development of this initiative as part of its Postdoctoral Junior Leader programme.

It is Colombo's aim to develop the mathematical tools required to improve the coordination and collaboration among dynamic robots. The main specific aim of the study is to design new algorithms for controlling decentralized formations, which will be applied to coordinating the movement of quadcoptors, or drones with four rotor blades. As this researcher explains: "Specifically, they will be capable of carrying out cooperative missions for the transportation of objects."



Leonardo Colombo

Mathematics to make driverless cars safer

Apart from the scientific and technological challenges posed by automatic driving in complex and unpredictable surroundings, scientists are also faced with other issues regarding its implementation, such as analysing the risks of this new type of automation, designing the communication between humans and machines and studying the impact it will have on the economy and certain industrial sectors, among others. This is the purpose of "<u>Trustonomy: the "Building Acceptance and Trust</u> in Autonomous Mobility" project in which ICMAT researcher and director of the <u>AXA-ICMAT Chair in Adversarial Risk Analysis</u>, Davis Ríos, is participating

This scheme has received 3.9 million de euros from the European Union H2020 programme. As its title indicates, the main aim of the project is to lay the foundations of trust and acceptance in automatic mobility. Ríos will be in charge of building risk analysis models capable of predicting and responding to the specific hazards arising from this emerging form of mobility. The project, which began on May 1st, 2019, is scheduled to be completed on April 30th, 2022, and will provide

improvements to the algorithms behind automatic driving. In addition to the ICMAT, 15 further organizatons from different European countries are also participating in the scheme.



The project Trustonomy will provide improvements to the algorithms behind automatic driving

A new European Union Marie Curie project for studying the Fourier transform

ICMAT researcher Javier Ramos has been awarded one of the European Union Marie Curie programme grants to study the so-called restriction of the Fourier transform, the purpose of which is to understand the behaviour of the Fourier transform restricted to null sets. Ramos analyses the particular case of functions that satisfy a series of properties known as L^p . His aim is to "determine for what values of p it is possible to restrict the Fourier transform of functions in L^p to certain types of surfaces". The restriction is in itself very surprising, because we cannot ensure continuity of the Fourier transform except for the case p=1.

In his project "A multilinear approach to the restriction problem with applications to geometric measure theory, the Schrödinger equation and inverse problems", this researcher intends to use multilinear estimates with optimal dependence on transversality in order to deduce linear estimates, which are a quantitative way of proving that the restriction is possible.



Javier Ramos

The theses completed at the ICMAT by M^a Ángeles García Ferrero and Carlos Mudarra are awarded two of the 2019 Vicent Caselles Prizes

Carlos Mudarra, a postdoctoral researcher at the ICMAT, and M^a Ángeles García Ferrero, a postdoctoral researcher at Max Planck Institute of Mathematics at Leipzig (Germany), are both former PhD students of the ICMAT. They have each been distinguished with a <u>Vicent Caselles Mathematical Research Prize 2019</u> from the Royal Spanish Mathematical Society (RSME) and the BBVA Foundation. Awarded to young researchers under 30 years of age who are in the initial stages



María Ángeles García Ferrero



of their careers, the aim of these prizes is to recognized young mathematicians "whose doctoral work breaks new ground and is influential in mathematical research at an international level, and to encourage them to continue along this path," as stated in the <u>BBVA Foundation citation</u>.

The jury charged with awarded the prizes remarked of García Ferrero, who completed her PhD under the supervision of Alberto Enciso (ICMAT-CSIC), that "her most outstanding results provide a full approximation theorem for solutions to parabolic equations and their application to the study of hot spots and isothermic surfaces". Of Mudarra, a former PhD student of Daniel Azagra (ICMAT-UCM), they state that "in his doctoral thesis he has solved problems regarding the approximation and differentiable extension of convex functions in Banach spaces".

ICMAT researcher David Pérez García awarded the Banco Sabadell Foundation Prize for Science and Engineering

The Banco Sabadell Foundation has awarded David Pérez García, a researcher at the ICMAT and a professor at the Complutense University of Madrid (UCM), with the 3rd Banco Sabadell Foundation Prize for Science and Engineering, in which the BIST (Barcelona Institute of Science and Technology) also collaborates. The jury, chaired by the economist Andreu Mas-Colell, awarded the prize in recognition of Pérez García's contribution to quantum information, Banach spaces and operator theory. Pérez García and his team work on the development of a new information theory, combining the ideas of the mathematician Shannon, on how to compress, correct and handle information by means of quantum physics. This prize is one of the most prestigious scientific awards for science in Spain. It is endowed with 50,000 euros and is awarded in recognition of the career achievements of young researchers.

mage: Universidad Complutense de Madrid



David Pérez García

Congresses, schools and thematic programmes

The ICMAT hosts the leading international experts on quantum computers

Quantum computers have ceased to be a promising technology of the future and are currently on the way to reaching concrete milestones. This new era in computing requires the establishment of groundbreaking physical and mathematical principles such as those being developed at the ICMAT Ignacio Cirac Laboratory. As part of this project, and with the collaboration of the Institutes of Theoretical Physics and Fundamental Physics and the University of the Basque Country, an international congress was held at the ICMAT between October 14th and 18th on Quantum Simulation and Computation (QSC2019) that brought together the leading world experts on the subject.

mage: ICMA1



John Martinis, head of the Google experimental group that is currently building a quantum computer, in his talk

Among the international speakers was John Martinis, head of the Google experimental group that is currently building a quantum computer. Also present was the coordinator of the OpenSuperQ project, Frank Wilhelm-Mauch, who aims to build a quantum computer in Europe using superconductor circuit technology. The QSC2019 provided the icing on the cake of this trimester that the ICMAT has devoted to quantum information theory. The main forerunner, Pérez-García, together with Ignacio Cirac, director of the Max Planck Institute of Quantum Optics and a Príncipe de Asturias prize-winner, is the coordinator of the ICMAT Laboratory that carries Cirac's name.

How are geometric methods applied to the natural sciences?

Both theoretical physics and engineering are fed by different geometric methods. This branch of mathematics has applications to quantum mechanics, relativity and cosmology, field theory, thermodynamics, classical mechanics, fluid mechanics and the numerical simulation of physical systems, among others. The ICMAT has a significant number of specialists in this field, who have come together to organize the thematic research programme "<u>Current Trends in Geometric Methods in Natural Sciences</u>" over a period of several months. The programme began on September 2nd, 2019 and will last until December 20th, 2019.

The programme includes three main activities: the <u>XXVIII Fall</u> <u>Workshop on Geometry and Physics</u> inaugural conference (held between September 10th-13th), the <u>GESTA School</u> (between September 10th and December 13th) and the <u>Workshop on</u> <u>Geometric Methods in Symplectic Topology</u> (between December 16th-20th), this latter being the closing event of the thematic trimester, in addition to a celebration of the 45th birthday of ICMAT researcher Francisco Presas, an expert in symplectic topology.



The thematic research programme "Current Trends in Geometric Methods in Natural Sciences" started with the 'XXVIII Fall Workshop on Geometry and Physics inaugural conference'



Banach Spaces: a key component in functional analysis

Held between September 9th-13th at the ICMAT, the "<u>Banach spaces</u> <u>and Banach Lattices</u>" congress brought together specialists from all over the world on Banach spaces, vector spaces with a norm for measuring vectors (and thus distances between elements in the space). Furthermore, a Banach space is also a complete vector space, which means that any infinite sequence of vectors always converges to a single limit within the space.

The congress consisted of four mini-courses, six plenary conferences, and daily short talks and poster sessions.



The "Banach spaces and Banach lattices" congress consisted of four minicourses, six plenary conferences, and daily short talks and poster sessions



The ICMAT hosted the first summer school devoted to fluid mechanics in June 2019

The future of fluid mechanics

In June 2019, the ICMAT hosted the first summer school devoted to fluid mechanics. International experts and 50 students interested in the field came to the centre to attend the event. Six 3-hour courses were given, during which subjects of great interest for current research were included, such as the study of weak solutions for Navier-Stokes equations, singularity and symmetry in two-dimensional incompressible fluid flow for these equations, and the loss of regularity in solutions to these equations.

The 2019 Escuela JAE de Matemáticas

Held at the ICMAT between June 10th-21st, the <u>Escuela JAE de</u> <u>Matemáticas 2019</u> (JAE Mathematics School) brought together 70 final-year degree and master students of mathematics, who learned about a series of topics that are frequently not included in academic curricula and are of particular interest in current research, such as geometric methods applied to robotics, the topology of complex manifolds, combinatorics and computerassisted proof applied to mathematical analysis.



JAE Mathematics School 2019

Outreach

How many multiplication methods do you know?

The latest book in the 'Miradas Matemáticas' collection, *Los* Secretos de la Multiplicación (The Secrets of Multiplication), is authored by Raúl Ibáñez, a professor at the University of the Basque Country and scientific popularizer. The book explores the history of humanity, particularly the history of mathematics, through the methods employed by different civilizations for the multiplication of numbers. The origins of this operation, which today is so simple and familiar, are to be found in prehistory, and its evolution is closely associated with systems of numeration.



The mathematician and scientific popularizer Raúl Ibáñez is the author of Los secretos de la multiplicación, ninth number of the collection 'Miradas Matemáticas'

In addition to many anecdotes and historical concepts, the book includes suggestion for putting the knowledge it presents into practice. As Ibáñez himself says: "It contains a series of didactical activities of a highly creative kind, with the aim of arriving at a better and deeper understanding of the subjects discussed in the text from other points of view, as well as learning more."

The 20th anniversary of the Ciencia en Acción scientific education contest

200 outreach experiences occupied the town of Alcoy in the province of Alicante between October 4th-6th as part of the 20th edition of Ciencia en Acción (CeA – Science in Action), a scientific education contest and fair in which the ICMAT has been a collaborator since its inception. "Over the years, I've been struck by the interest shown by teachers in getting students to participate actively in CeA," says Rosa María Ros, the director of the project since it began. "They've seen for themselves that this initiative is a melting pot for new scientific vocations. I still remember a remark made by a pupil at one of the first editions: 'We've discovered that we love discovering things.'"



The 20th edition of Ciencia en Acción (CeA – Science in Action), a scientific education contest and fair, was held in Alcoy (Alicante).

How many colours do you need to colour a map?

In conjunction with other centres belonging to the Consejo Superior de Investigaciones Científicas (Spanish National Research Council), the ICMAT celebrated the <u>Noche Europea de l@s Investigador@s</u> (European Researchers' Night) by means of a joint activity with Medialab-Prado (Madrid), which was held on September 27th. The Institute directed the "<u>Mapas y Mates</u>" ["Maps and Maths"] workshop in which was posed an apparently simple question, but one that has intrigued mathematicians for decades: the four-colour problem. This consists of finding the minimum number of colours that can be assigned to the countries on a political map so that no two countries sharing a common boundary have the same colour. Graph theory, topology, computer-assisted demonstrations and other concepts related to this mathematical problem were addressed during the workshop.

Jezabel Curbelo, a professor at the Autonomous University of Madrid and an ICMAT member, and Marco Castrillón, a professor at the Complutense University of Madrid, were in charge of conducting this activity, with the support of the ICMAT Unit of Scientific Culture.



The Institute directed the "Maps and Maths" workshop in the European Researchers' Night

The episode "Limits of Mathematics" concludes the first season of 'Revoluciones Matemáticas'

The fifth episode in the cartoon series '<u>Revoluciones</u> <u>Matemáticas</u>' ['Mathematical Revolutions'], entitled "<u>The</u> <u>Limits of Mathematics</u>", tells an informal, exciting and amusing story about the history of mathematics and its revolutionary consequence: the incompleteness of mathematics. In this episode created by the ICMAT, the Divermates company and animator Irene López, the famous theorem devised by Austrian mathematician Kurt Gödel is illustrated by using a chess board. In addition, an <u>activity</u> is proposed to explain the concept of a proof; that is, the sequence of logical steps to be followed in order to establish a valid mathematical reasoning.

The first season of the series drew to a close with this episode, although a second season that forms part of a project called *Ciudad Ciencia* (Science City) under the auspices of the CSIC Vice-Presidency of Scientific Culture is already underway and will be released during the current course.



The first season of the series 'Revoluciones Matemáticas' drew to a close with this episode





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