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EDITORIAL

Challenges of Mathematical Biology

Andreas Deutsch (TU Dresden). A hiking mathematical biologist encounters a shepherd. The shepherd is so happy that he offers the mathematical biologist one of his sheep should he be able to calculate the exact number of individuals in his herd. The mathematical biologist estimates the number to perfection. Later one can watch the mathematical biologist hiking further together with the shepherd's dog. This joke nicely expresses the essence of mathematical biology but also the danger of mathematical abstraction. The strength of mathematical biology is not just to calculate quantities but to abstract common patterns at different biological organisation levels which is exaggerated by the mathematical biologist above. The interface between the biological problem and the mathematical analysis is the model. Mathematical biology fosters mathematical reasoning to the explanation of biological phenomena, and can provoke new types of mathematical problems. Thus, ecological interactions have motivated new questions in mathematical game theory, and properties of cancer cells have triggered the study of density-dependent diffusion equations.

Today mathematical biology bustles with activity. Under the new umbrella of systems biology and medicine it offers promising perspectives for mathematicians, physicists, and theoretically interested biologists. Many new institutes, curricula, and research networks in systems biology and medicine manifest this boom. However, a boom raises suspicion and might offer only short-term perspectives. So the question is: is there also a long-term perspective?

First of all, mathematical biology is nothing new! There are Baltic, British, French, German, Italian, Spanish, and Russian roots ranging back to the end of the 19th century. Mathematical biology was initially triggered by enormous amounts of ''data'' arising from new observations (e.g. from expeditions into colonial countries) and new experimental techniques. The famous Mendel experiments and a fertile communication between experimental biologists and applied mathematicians in the 1930's marked the beginnings of population genetics and created a nucleus for mathematical biology.

Responsible for the recently experienced boom of systems biology is again a data jungle, now called big data and predominantly stemming from new molecular biological methods together with the concurrent rapid development of high performance computing capabilities. As the 20th century was the ''century of physics'', the 21st century will be the ''century of biology and medicine'', and the essential developments will only be possible from strong support by mathematical and computational methods at the data acquisition, representation and interpretation level. Together with specialists from related disciplines as bioinformatics and biophysics, mathematical methods are applied to more and more biological systems and medical problems. Rather new and hot applications are in the fields of bioengineering, biological regeneration, immunology, infectious diseases or personalised tumour therapy. Besides the need for developing models for particular biological and medical problems, mathematical biology helps to extract general principles of complex systems. A better understanding of such principles can be further exploited as bio-inspired solutions for technological applications.

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Personally, I had been interested already during school days in mathematics AND biology. So what should I do? I consulted a professor of biology who advised me to concentrate on a full mathematical study and subsequently get involved in biology, for example through a PhD. This turned out to be a very good advice which is applicable still today. While biology has a modular structure - the different fields as cell biology, evolution or ecology can be studied quite independently of each other – the mathematical curriculum follows a sequential order. One needs an introduction to the basics as analysis, linear algebra and numerics before one can proceed with advanced topics like differential equations, or stochastics. During my PhD it became clear that doing 'wet experiments' and 'dry modelling' together was not a long-term perspective because both aspects are highly demanding. Today, I am head of the department ''Methods of Innovative Computing'' at the Centre for High Performance Computing at Dresden University of Technology. My research group develops and analyses mathematical models, algorithms and simulations in the dry-lab, but in very close cooperation with biological and medical wet-labs.

Given the enormous amount of biological and medical data that already exist and that can be expected there seems to be enough to do for mathematical biologists in the coming decades. However, having to decide for a personal career direction I would not recommend a 'data-driven' but 'interest-driven' decision. For example, early in my career I started to develop cellular automaton models although there was not much biological data available for this kind of model at that time. Today the data are there and can now be successfully analysed with the cellular automaton models developed long before (see A. Deutsch, S. Dormann, Cellular Automaton Modeling of Biological Pattern Formation, Birkhauser,Boston, 2018).

Further information on conferences, curricula, schools, and open positions can be found at the websites of the two leading societies in the field, the European Society for Mathematical and Theoretical Biology (www.esmtb.org) and the Society for Mathematical Biology (www.smb.org). A key activity within the Year of Mathematical Biology is the 11th European Conference on Mathematical and Theoretical Biology, Lisbon, July 23 – 27, 2018.

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More info: https://tu-dresden.de/zih/die-einrichtung/struktur/andreas-deutsch

Image: Sergio Martínez of Austin, US

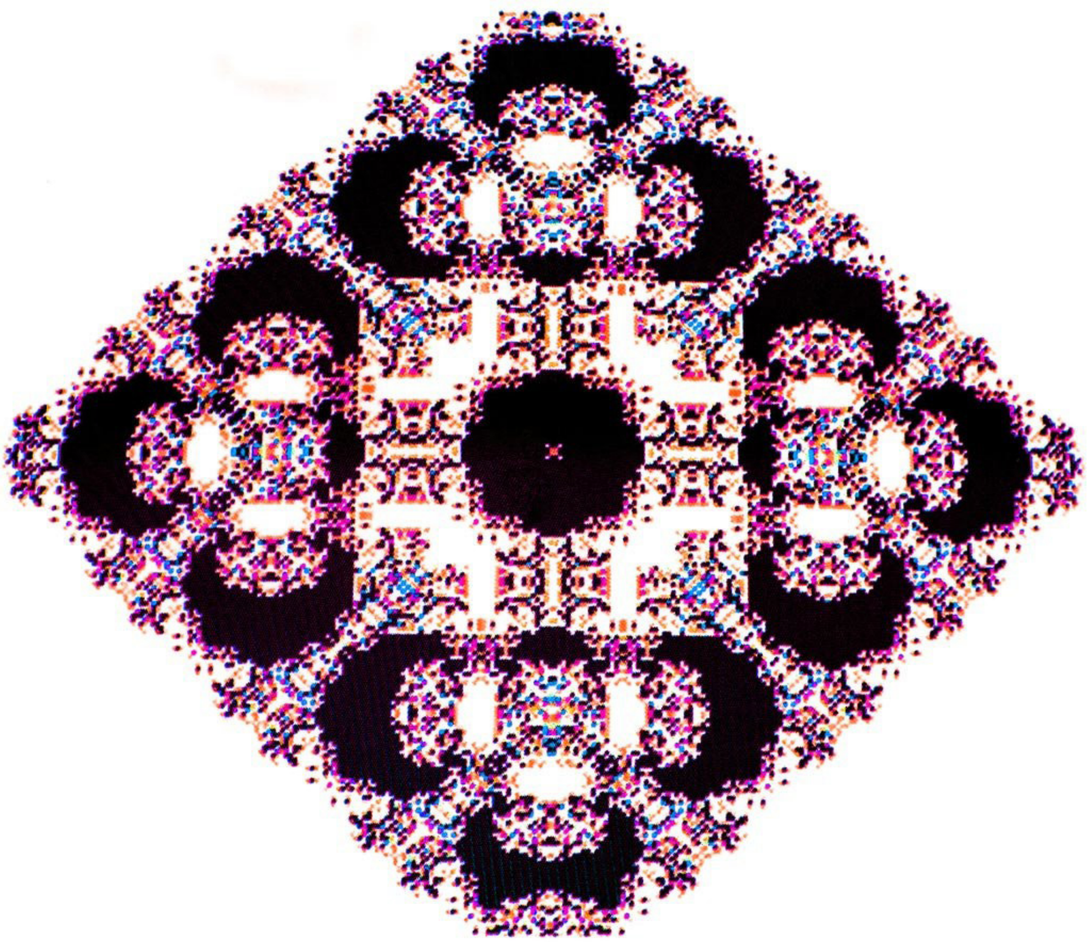

Image of a cellular automaton.

INTERVIEW: José Antonio Carrillo, researcher at Imperial College London, and until December 2017 chairperson of the European Mathematical Society (EMS) Committee of Applied Mathematics

"For mathematics, biology is now what physics was in the early 19th century, an inexhaustible source of interesting problems"

Image: José Antonio Carrillo

José Antonio Carrillo (Imperial College of London) has been one of the promoters of the celebration in 2018 of the Year of Mathematical Biology.

José Antonio Carrillo is one of the instigators for the year 2018 being declared as the Year of Mathematical Biology, a joint initiative by the European Society for Mathematical and Theoretical Biology (ESMTB) and the European Mathematical Society (EMS), where in the latter institution he was the chairperson of the Committee of Applied Mathematics between 2014 and 2017. It is no coincidence that his field of research is in partial differential equations, of which he says that "they're one of the most important mathematical tools for the modelling of life and the socio-economic sciences".

Since 2012, Carrillo has held a chair in Applied and Numerical Analysis at Imperial College in London. Prior to that, from 1998 to 2000, he was Research Professor with the *Institución Catalana de Investigación y Estudios Avanzados* (ICREA) at the Autonomous University of Barcelona. During this same period he was a professor at the University of Texas in Austin (USA), and before that he had held positions of associate and tenured professor at the University of Granada, where he also completed his PhD.

Elvira del Pozo

The idea of advocating the role of mathematics in biology was in part due to you. Why is that?

Yes, I was one of the forerunners during my time on the EMS Committee of Applied Mathematics. I and other colleagues put forward the idea of organizing some kind of activity as a result of the growing interest in the area of mathematics that seeks to solve problems arising in biology. So we agreed with the European Society for Mathematical and Theoretical Biology (ESMTB), which is the particular mathematical society focusing on biology, to organize jointly an activity that, on the one hand, would spotlight what had been achieved so far in this field, and on the other spark a debate to identify the challenges for the coming years.

And after a year, "that activity" became a single theme devoted to this discipline. Why is it so important?

For mathematics, biology is now what physics was in the early 19th century, an incentive, an inexhaustible source of interesting problems. And while it's not exactly new, because for more than a century mathematical models have been used, for example, in population dynamics, recently the field has become more extended. Models are becoming increasingly more sophisticated in the attempt to explain a range of phenomena, from microscopic biological processes such as the interaction among cells of an organism, to more macroscopic processes in which explanation is sought for the global behaviour of a system. All of this is leading to many interesting developments and new applications.

Why is this boom taking place now?

The impact of the development of computational capacity and the use of sophisticated numerical data in complicated problems where the amount of variables is immense is quite clear. What's more, questions are being asked in biology that demand the development of a fresh applied mathematics, completely different from that traditionally required by physics.

"The models are becoming increasingly sophisticated; they try to explain a range of things from microscopic biological processes, such as the interaction among cells of an organism, to macroscopic processes"

If biology *inspires* mathematics, what is the function of mathematics exactly?

Mathematics enables the validation of working hypotheses. The methodological rigour of mathematics is infinitely valuable for ruling out or endorsing assumptions made by biologists, neuroscientists and medical researchers. In a way, it's the mathematization of biology. Based on observations and experiments, mathematicians try to create a model capable of explaining a phenomenon. The first one they come up with may be quite removed from reality, but little by little improvements are made and variables introduced that were not taken into account before. So by asking new questions they are able to build a model that describes the observed phenomenon as faithfully as possible. In a way, that's how modelling also began in physics, in continuum mechanics.

What type of mathematics is employed?

That depends on the type of question researchers ask themselves. Differential equations and partial differental equations are used, as well as differential equations with noise, which are what I'm working on, but there are a multitude of mathematical technologies.

A successful example is computational neuroscience, which is experiencing a great development.

That's right. In fact, I'm acquainted with the research work being done at the *Centre de Recerca Matemàtica* (CRM) in Barcelona. The team is very multidisciplinary, with physicists, neuroscientists, biologists and mathematicians who are trying to explain the activity of neurons in the brain. There are many very sophisticated models that attempt to describe the formation of the electrical signals emitted by a neuron. Others are designed to determine the activity in a certain part of the brain, and ultimately, to determine how a set that may contain 10^9 neurons behaves. To solve this type of problem you need a model with at least 10^9 sub-models for each neuron. The complexity is enormous. That's why simplified mathematical models have been developed on the basis of real microscopic models of neuron behaviour, which are studied to see if particular patterns of neuron activity exist.

The research work on certain mechanisms of the brain was awarded the 2014 Nobel Prize for Medicine.

That's an example coming from the work on computational neuroscience in which John O´Keefe, Edvard I. Moser and May-Britt Moser developed models based on systems of differential equations and arising from real experiments, with the object of understanding how we get our bearings in space. They realized that laboratory rats were able to reconstruct their movement in complete darkness. How were they able to do that without any reference point? They discovered that a part of the rodent brain possesses a type of neuron that produces electrical signals capable of identifying particular locations. This enabled the researchers to trace a kind of map of inter-nodal movement, and they believe that the same mechanism exists in human beings, although they haven't been able to prove it yet.

Part of your research is devoted to this field.

Yes, at the moment I'm working in computational mathematics on something that the Norwegians Edvard I. Moser and May-Britt-Moser are working with; models of stochastic differential equations applied to the activity of neural networks.

You've also been working on other aspects of mathematical biology.

Yes, specifically, a very particular aspect of cellular movement – chemotaxis – the phenomenon by which bacteria and other cells of organisms direct their movements according to the concentration of certain chemical substances in the environment. Highly complex mathematical models exist that enable typical behaviours to be understood. I've also been involved in collective behaviour models. For example, why do shoals of fish or flocks of birds swarm together the way they do? How do they manage to move in this coordinated way? We're trying to describe these behaviours with basic models in which hypotheses are made about how individuals interact with each other. For instance, the behaviour of pedestrians in the *calle Preciados*, the street here in the center of Madrid, over Christmas. People try not to bump into each other, to steer around the others, but since people are social beings they tend to move in groups and have an attraction to certain individuals. They also tend to coordinate themselves with the rest, by following the person in front or naturally forming two lanes, one in each direction.

"The rigour of mathematics is very valuable for ruling out or endorsing the assumptions of other scientists"

What other fields in biology could benefit from the interaction with mathematics?

Another interesting field is cancer prediction, which is currently on the rise. A lot of people, both in Spain and in Europe, are working on models of cancer cell behaviour. They provide valuable information for chemotherapy and other types of effective treatment.

What about climatology, ecology, population dynamics...?

Mathematics plays an important role in all those fields. In fact, population dynamics is one of the areas with the most mathematical tradition, and the one that gave rise to what we now call modern mathematical biology. Outstanding groups are working on this subject in Spain.

"From cursed to coveted"

In recent years, mathematics has become one of the favourite subjects of university students

Image: BBVA Foundation

More and more students are choosing Mathematics as a university career.

Ignacio Fernández Bayo. "I've liked mathematics since I was a little girl, maybe because of my brother's influence. He's ten years older than me and a mathematician, but I've always enjoyed solving problems," says Ángela Capel, who is currently doing her PhD thesis at the Institute of Mathematical Sciences (ICMAT). Like her colleague, David Alfaya, Ángela is a typical example of the traditional student of mathematics, someone who loves the science of numbers and has a clear vocation. They belong to a minority group of people that stands in stark contrast to the majority of secondary school students who find the *cursed* mathematics hard to swallow. This minority is so small that for years the popularity of this subject was so low that the pass mark for access to study it at university was also very low – five out of ten. At the moment, and for a few years now, things have changed completely and some branches of the degree even top the ranking of those most difficult to access.

According to Margarita Arias, a professor at the University of Granada and head of the secretariat for the Deans of Mathematics Conference: "We've moved from always having vacant places to an enrollment of 90 percent or even higher, and this is happening in all the universities." She goes on to say that: "I think that the policy of dual degree courses has really worked out well, for Informatics and Mathematics, and for Physics and Mathematics, which has the highest cut-off mark (13.8 out of 14)."

Dual degree courses have resolved the dilemma that confronted many students when they hesitated before the choice between Physics or Mathematics. This is the case of Patricia Contreras, currently engaged on her PhD at the ICMAT. "I did a double degree in England because I liked both sciences. If I could, I would have taken a quadruple degree course." Like her, many students who stood undecided on the border of the two sciences have been able to fulfill their ambitions by taking a *two-in-one* degree course. Her colleague Roi Naveiro followed a more complex path: "Influenced by my father, I started studying medicine, but later changed to Physics and completed a Master in Theoeretical Physics. From there I switched to Mathematics, because it provided more career options."

In the end, this rise in demand and popularity can largely be explained by the many professional opportunities opened up by qualifications in these subjects. The use of mathematics in all types of spheres and activities has become increasingly evident, and everything points to the fact that people with qualifications in this discipline find it easier to obtain employment in many sectors. "I've been teaching in secondary schools for 30 years and have always seen a small minority of pupils who like mathematics above all other subjects. Then there are those who have a scientific vocation and choose a university course because it will provide them with many options for a professional career. When things were looking good, engineering studies attracted most of those students, but in recent years they've found it much more difficult to find good jobs, except in aeronautics, and many of them have had to go abroad to seek employment, while right now mathematics is opening the door to professional opportunities they didn't have before," says Onofre Monzó, president of the Federation of Societies of Mathematics Teachers.

Traditionally it has been said that three types of secondary school pupils exist: those who love mathematics, who have always formed and continue to form a minority; those who detest mathematics, who form a big majority, and those who are somewhere in between. And it is in this last group where changes have taken place. "Previously, a mathematics degree was very hard and only led to two professional choices; either opt for research and try to obtain a job at a university, or go into teaching in a secondary school. Now, those with higher education qualifications in mathematics go into industry or the service sector," adds Monzó.

However, this shift in perspective has its downside. As Margarita Arias explains: "As the range of possibilities has become wider, the number of those who want to teach in secondary schools has fallen, and teaching associations are asking us to promote the vocation to teach." A further drawback is that students often find employment before leaving university, which is what happened before in engineering, and many fail to complete the course.

The financial sector was one of the first to employ mathematicians, and the experience appears to have been a good one, because vacancies in areas like market analysis and investment, mathematical modelling and jobs in the banking sector have not stopped growing. Another sector on the rise and in significant need of mathematicians is big data, the application of which is spead over many sectors of industry, services, commerce and communications. A dual degree in Mathematics and Informatics strongly favours professional opportunities in new technologies, which involve practically all the economic and financial sectors.

On the other hand, students who opt for a dual degree with Physics or Mathematics are invariably drawn to an academic or research career. They are the ones who often choose to do a PhD after they complete their degree, as in the case of Ángela, David, Roi and Patricia. "I was always sure that I wanted to do research," says David Alfaya. "I'm passionate about abstract thought and problem-solving." However, having a vocation is not enough on its own, because the fact of the matter is that the number of posts in research has virtually remained the same. After seven years concatenating grants, David is about to complete his thesis and is looking for somewhere to do his postdoc. "There are very few opportunities in Spain, but anyway it's better to do it abroad. Even so, there's a lot of competition. In France there are one hundred candidates for each position".

There is no doubt about Ángela Capel's vocation for research either. "I was undecided between Physics and Mathematics because I loved both subjects, and at that time dual degrees didn't exist." In her first years at university she studied both subjects concurrently until she eventually opted for Mathematics. With eighteen months to go before she completes her PhD, she faces the same uncertainty as David about her future, due to the scarcity of available posts. "I'm still hopeful about being able to continue doing research, but I don't know if that will be possible, so I'm prepared to consider other openings, either in industry or in teaching, even though my sense of vocation is not so strong there."

Image: BBVA Foundation

Young PhD students during the BYMAT congress.

Patricia Contreras is less sure about what she wants to do in the future. She pursued the idea of studying the Philosophy of Physics and did a Master at Oxford. However, Brexit and feelings of homesickness brought her back to Spain, where she looked for and eventually found a research group headed by David Pérez (Complutense University-ICMAT), where she was able do her thesis. "I was lucky because I managed to get an FPI grant. I've completed three years and still have two to go. I began by wanting to do research come what may, but now there are other professional opportunties in view. The fact is that I still don't know what I'll do when I finish my doctorate."

As for Roi Naveiro, he has always had a foot in both camps and is not about to rule out any possibility. "I'm open to everything, and when I finish my thesis in two years time I'll see what my options are. I'm lucky in the sense that I usually like the work I do." He has crossed over from physics to mathematics once already, and it would not be difficult for him to do it again in reverse. In any event, he says that he likes research but would be perfectly prepared to consider something else.

These different job prospects also have an influence on the places provided by universities. As Margarita Arias says: "In my personal opinion, Physics and Mathematics are subjects that lend themselves more to research, so the number of students who study them are smaller – on average about twenty places – while here at the University of Granada we provide 50 places for Informatics and Mathematics and 100 for pure Mathematics."

The increase in demand for places has made the minimum grade for access higher, but according to Arias this higher grade has also led to an increase in demand because it has conferred prestige on the study of Mathematics. "It's acted like a pull effect on many parents. When they saw that the minimum entrance grade for a degree course in Mathematics was 5 out of 10, they tended to think their kids would be wasting their efforts. Now they believe the opposite."

The presence of the discipline in the media has also contributed to its rise in popularity. As Arias says: "I think a lot of good work has been done to promote mathematics, and now it's easy to find news items about it in the media, when a few years ago that was not the case. This exposure has changed attitudes towards Mathematics by revealing it as something of great practical use."

Another factor that has made Mathematics a coveted object of study are the initiatives being taken at a secondary school level. Perhaps it is no coincidence that David and Ángela, who say their mathematical vocation is much clearer to them than to Patricia and Roi, have had experiences that contribute to awakening this vocation, such as the Stimulus for Mathematical Talent scheme (Estalmat) and the Mathematical Olympiads. According to Monzó, who collaborates with Estalmat in Valencia, it is evident that "the level of knowledge and achievement in Mathematics is improving because these secondary school activities involve fun, visualization and outreach rather than just study per se. The Olympiads provide the opportunity to create groups and teams as part of their preparation, and those who get through to the final are not just good but excellent."

For some secondary school pupils, mathematics is still a subject they flee from like the bogeyman, but more and more students are discovering the magic of the science of numbers and logical thought in a society that increasingly values the discipline and those who practise it. The upsurge in job prospects arising from the study of this subject is the icing on the cake of the conversion undergone by mathematics from cursed to coveted.

Image: BBVA Foundation

From left to right: David Alfaya, Ángela Capel, Roi Naveiro, Patricia Contreras and Jesús Ocáriz, the ICMAT students who organized the BYMAT congress.

BYMAT, young mathematicians in search of a future

In May of this year, Ángela Capel, Patricia Contreras and Roi Naveiro, together with their colleagues Jesús Ocáriz and David Alfaya, all doctoral students at the ICMAT, organized a meeting to debate the future prospects of recently graduated and doctoral students. "It all began after a stay I had in Paris, where PhD, postdoc and senior students met together every afternoon to get to know each other, exchange opinions and talk about job prospects over a cup of coffee. I made a lot of contacts there, and when I came back to Madrid I suggested the idea of doing something similar," says Ángela. And Patricia adds that "we didn't know many people doing doctorates, either in Madrid or in the rest of Spain, and sometimes not even the people in the office next door. Obviously there was something wrong. What's more, we realized that we couldn't all get jobs in research and wanted to find out what the alternatives might be."

That is how "Bringing Young Mathematicians Together" (BYMAT) came into being. In spite of its modest beginnings, the underlying drive of the idea was obviously much broader and more important, because its appeal grew nationally and then extended to other countries, until it eventually reached proportions that surprised its initiators. Help from the ICMAT and the BBVA Foundation, which provided financial backing, enabled the meeting to exceed the initial limits with the participation of 181 people from 78 institutions (38 from abroad) and 13 different countries including Morocco, India, Ghana, Brazil and Mexico, among others. Over a three-day period between May 7th and May 9th, those who attended took part in discussions and listened to many speakers who talked about the different professional opportunies currently open to mathematicians.

"The idea was to organize a comprehensive conference with different keynote speakers from diverse backgrounds as well as workshops and round-table discussions. These speakers came from many different sectors," says David. The subject of scientific communciation and outreach was also discussed, adds Roi: "We must remind society of our existence and the importance of mathematics," while Patricia points out that "society ought to know that what we do is solve problems."

The success of the conference has led to its continuation. Preparations for BYMAT 2 are already under way as a space for sharing problems and discussing how to bring mathematics closer to society, as well as addressing other issues. As Roi explains: "Now the ideas are going to come from people themselves. We're going to provide the initial impetus, but others will also make their contribution." Furthermore, a BYMAT network open to all young mathematicians is being set up to ensure the future of the initiative, a broadly defined concept, since as Patricia remarks: "It's not about age groups, but rather an academic issue. We believe that students and postdocs are faced with common issues that do not affect tenure-track professors."

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ARTICLE: The mathematics of the immune system

T-lymphocytes are cells that form part of the immune system in the human body. Their processes of production and maturation are particularly delicate, since the slightest failure can lead to serious medical problems, such as leukemia and other autoimmune diseases. In recent years, differential equations have played a key role in the mathematical models employed for studying populations, and these models can also be used for understanding T-cell mechanisms.

Carmen Molina París and Ágata Timón. T-lymphocytes form part of the *adaptive immune* response, the second stage triggered by the immune system for the protection of the organism against infections caused by viruses, bacteria and all kinds of pathogens. Prior to that, cells (such as the macrophages and neutrophils) and molecules (such as the interferons) found in the mucous membranes and the skin, the tissues most exposed to infection, provide a non-specific immune response; that is, more or less independently of the pathogen causing the infection.

In the second stage, the T-lymphocytes send an alarm signal to the B-lymphocytes (capable of secreting antobodies) and kill the cells infected by the pathogen causing the infection. T-lymphocytes are found in the blood, the lymph and the lymphic nodes and are in a constant state of circulation in order to carry out their task of surveillance. They are produced in the bone marrow from hemotopoietic stem cells which mature to become precursors of T-lymphocytes by means of *thymic selection*, a process of cellular differentiation that lasts approximately three weeks and takes place in the thymus (lymphoid organ). This thymic maturation involves a series of molecules and other cells (epithelial cells of the thymus) that send signals to the pre-T lymphocytes or thymocytes.

Errors ocurring in thymic selection may lead to types of leukemia, autoimmune diseases or immune insufficiency, so it is important to understand what molecular or cellular mechanisms are involved. To this end, scientists use mathematical population models to simulate the process and obtain information in a non-invasive way.

These models describe the evolution of each thymocyte throughout the process of maturation. At any given moment a cell may (1) die, (2) divide and give rise to two daughter cells, or (3) differentiate and yield a different cell. It is extremely important to understand both the kinetics of this process and where and when each thymocyte receives a signal that indicates the path it must follow. These signals depend on the epithelial cells of the thymus, especially the types of molecule (antigens) they carry in their cellular membrane, as well as the type of T-receptor that the thymocyte displays on its surface. It is precisely the interaction between the T-receptors of a thymocyte and the antigens of the epithelial cells that determines the future of the thymocyte in question.

If the interaction is of great biochemical affinity, the thymocyte will die from apoptosis (programmed cell death); if this affinity is very small or nonexistent, death occurs by negligence, and if affinity is of an intermediary level, the thymocyte undergoes a process of differentiation and maturation continues. In order to quantify the kinetics of thymic selection, it is necessary to introduce into the model death rates (the frequency with which a thymocyte receives a death signal) and differentiation or proliferation rates (the frequency with which a thymocyte receives a differentiation or cellular division signal).

Knowledge of these rates would enable the determination, for example, of the average time that a thymocyte spends in each phase of the thymic maturation process. However, it is not possible to determine this experimentally, since it would require observation of the path of each pre-T lymphocyte in the thymus, and current microscopic techniques, such as two-photon imaging, only enable this to be done for an hour at most, which is a period of time much shorter than the time scale of the thymic process (several days).

Mathematics provides precise tools for describing cell populations and their changes over time by means of deterministic population models. In essence, these models describe the evolution of populations over time. If one assumes that a population is initially composed of a certain number of individuals, the equation describes how many individuals there will be shortly afterwards and whether or not the population has changed due to migration, death, or birth of new individuals. Each population model will depend on what is defined as the migration mechanism (for instance, the constant flow or otherwise of individuals) and the death and birth mechanisms.

Ordinary Differential Equations (ODEs) enable the number of thymocytes that exist at each stage of maturation to be described over time. Mathematical population models include the death, differentiation and division rates that characterize the process of thymic selection.

We consider the basic model of the figure, which describes four thymocyte populations, from the first stage to the last. The parameters required are as follows: the incoming flow from the bone marrow to the thymus (φ); the death rate of each population (μ); the differentiation rate (ϕ), the proliferation rate (λ), and the migration rate to the blood of the lymphocytes that have survived all the process (ξ).

In this model it is assumed that the process of maturation in the thymus is as follows: the thymocyte population in the pre-DP (population 1) stage receives a flow of stem cells φ. These thymocytes have two possible destinations: population differentiation post-DP (population 2) (at a rate of ϕ_1 per cell) or death (at a rate of μ_1 per cell). The post-DP thymocytes have three possible destinations: population differentiation CD4 SP (population 4) (at a rate of ϕ_4 per cell), population differentiation CD8 SP (population 8) (at a rate of ϕ_8 per cell) or death (at a rate of μ_2 per cell). Finally, the CD4 or CD8 SP thymocytes have three possible destinations: maturity and migration from the thymus to the blood (at a rate of ξ_4 or ξ_8 per cell, respectively), death (at a rate of μ_4 or μ_8 per cell), or division (at a rate of λ_4 or λ_8 per cell).

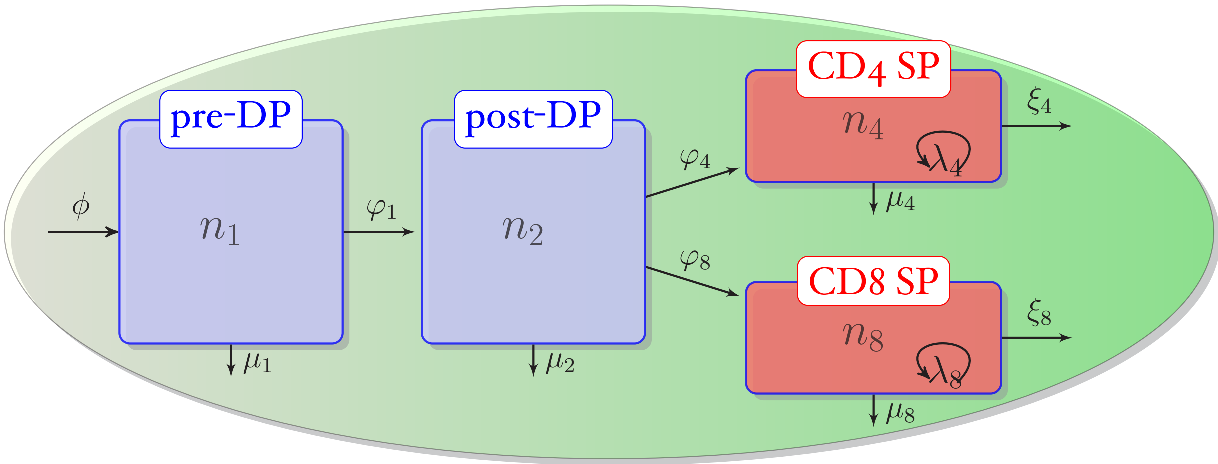


Image: University of Leeds

The model, developed by Carmen Molina París, describes the evolution of four populations of thymocytes.

The experimental data required by the model to determine the parameters are the number of thymocytes in each population at different moments in time. The virtues of the model are its simplicity, its direct relation with the experimentally determined variables (the number of cells in each population), and the fact that from a mathematical point of view these models have been studied extensively. The limitations of the model arise likewise from its very simplicity; it is assumed that the process is irreversible, deterministic and non-random. Furthermore, all the thymocytes behave in an identical manner; that is, the model does not take into account any possible cellular heterogeneity, although we know that it does exist.

In collaboration with Kris Hogquist, a professor at the University of Minnesota and an expert in thymic development, we have developed a research line in the study of thymic selection in rats. Combining her experimental data with our basic model, we have determined that 66% of the thymocytes belonging to population 1 (pre-DP) die from negligence; 92% of the thymocytes in population 2 (post-DP) die from apoptosis (programmed death); 5% move to the CD4 SP phase and 3% to the CD8 SP phase. We have also been able to estimate that 46% of thymocytes from the CD4 SP phase (population 4) either divide once before death or migrate from the thymus to the blood. In the case of the thymocytes in the CD8 SP stage, this figure is reduced to 27%. Our study concludes that fewer than 9% of the pre-T lymphocytes that start thymic maturation manage to complete the process. However, this mathematical model does not take into account the possible stochastic effects or the cellular heterogeneity that characterize all biological processes.

The above is just one of the many examples of mathematics applied to immunology. Other applications include the study of T-lymphocyte populations during an immune response to a virus or to tumours. There still remains much ground to cover; a current major challenge is to gain an understanding of both the dynamics and the molecular mechanisms involved in the immune response to tumours, and thereby improve the existing immune therapies for the treatment of cancer. An important challenge for mathematicians is how to describe biological processes in terms of individual cells (rather than in terms of populations), and thus be able to model the heterogeneity of each cell in a population of interest.

Image: University of Leeds

***Carmen Molina París* is a professor at the Leeds University Department of Applied Mathematics (United Kingdom). She gained her degree in Theoretical Physics at the University of Granada and completed her PhD in Quantum Field Theory under the supervision of Bryce DeWitt at the University of Texas Center for Relativity in Austin (USA). After a 3-year post-doctoral stay at the Los Álamos National Laboratory, she returned to Spain in the year 2000. Somewhat by chance, she arrived at Warwick University in October, 2001, where she was introduced to mathematics and immunology. In September, 2002, by which time she was a tenured lecturer at the Leeds University School of Mathematics, she formed a new research group devoted to Mathematical Immunology. In close collaboration with immunologists, she is currently engaged in the development of mathematical models to further the understanding of the molecular and cellular mechanisms that give rise to immune responses. Markov stochastic processes are en extremely useful tool for describing the heterogeneity and the evolution over time of different populations of immune cells. She has been the coordinator of three European projects belonging to the FP7 programme, and from September, 2018, she will be the coordinator of the Marie Skłodowska Curie innovative training network in Quantitative T-cell Immunology and Immunotherapy (Quan TII).**

ICMAT
INSTITUTO DE CIENCIAS MATEMÁTICAS

ICMAT QUESTIONNAIRE: Benoit Perthame (Sorbonne University)

"Modelling can explain the observations made by biologists"

Image: Benoit Perthame

Benoit Perthame (Sorbonne University) tries to answer questions that arise in the life sciences, using mathematical tools.

Benoit Perthame was born in France on June 23rd, 1959. He studied for a mathematical degree at Ecole Normale Superieure. In 1982, he obtained his PhD from the University of Paris Dauphine, under the supervision of Pierre-Louis Lions. Currently, he is Professor of applied mathematics at Sorbonne-Universite.

Ágata Timón

Why did you choose mathematics ahead of any other subject?

I have been hesitating between mathematics and physics, but I found mathematics more attractive, mainly because of my teachers and also because it was providing the tools to understand physics.

Besides mathematics, which activities do you like most?

Who said 'My main preoccupation is to keep some time from my research for my students, and to keep some time from my students for my family' ?

A movie, book or play you'd recommend?

The book "Guns, germs and steel" by Jared Diamond.

How was your first encounter with mathematical research? How did you became interested in biology? How did you find out that these two fields can be combined?

I began my research in the field of stochastic control for a master thesis and then PhD, the field is vast and was emerging with the notion of viscosity solutions, it was a very active time for that field with many new ideas. After that I turned to questions motivated by kinetic physics (plasmas, fluids) and multi-scale analysis. At the end of the 1990's I realized that there were many teams involved in all areas of physics but very few in problems of biology. That is why I decided to investigate the modeling questions in the various fields of life sciences.

What did you like most about your early experiences with math-biology research?

I discovered that the way to think in biology is very different from physics. Models are not well established, the mathematics should tell you about the qualitative behaviours rather than exact numbers, coefficients are not fixed (adaptation of organisms is important). Behind these questions various theories emerge as pattern formation, waves, uncertainty quantification, instabilities and asymptotic theory because one always learns more from extreme cases than from the normal behaviour.

How is the relationship between mathematicians and biologist? How do they understand each other?

Most of the biologists are experimentalists whose goal is to discover new phenomena or new observations. In a second step, when it comes to explain these observations, when they are confirmed by several experiments in different conditions, modeling is usually welcome.

There is also a recent trend to organize teams of physicists and biologists working on living matter. Then, the contact is much easier.

Which scientist impressed you most during your career?

I often say that there are two remarkable results about Partial Differential Equations in the first half of 20th century. The Lax-Milgram theorem tells you that to theoretically solve an elliptic PDE is to build a Hilbert space, and to numerically solve it, you need to fill well that Hilbert space in finite dimension. The Turing instability mechanism tells you that diffusion can destabilize a stable dynamical system.

If you could have a one hour blackboard discussion with an ancient scientist, whom would you choose to meet and what would you discuss?

Being head of my laboratory, I need some pragmatism and organize discussions in advance, but only when they are relevant.

Do you have a particular theorem or formula used in any field of biology you especially like?

I like the equations for neurones by Catherine Morris and Harold Lecar. It gives a remarkably simple structure which, if seen abstractly, contains many other models.

What is your favourite math-biology book?

Jim Murray's 'Mathematical biology' is remarkable. In a different style, the book by J. Jost, 'mathematical methods in biology and neurobiology' is also remarkable.

How would you describe/sketch your research interest in a few lines?

To discover what is the mathematical interest behind the questions arising in the life sciences.

Which recent results in your field would you highlight?

I like particularly the vast activity that has been developed recently around asymptotic methods to describe evolution based on mutations and selection processes, from stochastic agent based models to the population point of view. This topic is relevant in all biology and mathematically deep and diversified.

Which particular math-biological problem do you consider especially challenging?

To understand the mathematical structures and solutions behind the various approaches by neuro-physicists on neural assemblies, mean field models, learning processes.

Which subjects in math biology outside your field would you like to learn more about?

Developmental biology: combining genes, growth, geometry and mechanics.

In the future, where do you think the interaction between mathematics and biology may be more fruitful?

It seems to me that the present questions in biology, based on present observations, have to do with molecules and the way they organize the cell dynamics and behaviour in its environment.

This year is the International Mathematical Biology Year; what do you think is the main usefulness of this celebration?

It shows that the mathematical community has now accepted the idea that biology also yields challenging new mathematical questions.

TELL ME ABOUT YOUR THESIS: Beatriz Pascual Escudero

Image: Beatriz Pascual

Beatriz Pascual Escudero investigates possible connections between invariants of singularities that arise in terms of the space of arcs of a variety and the information (the invariants) that is usually used to define resolution algorithms.

Title: "Algorithmic Resolution of Singularities and Nash Multiplicity Sequences".

Author: Beatriz Pascual Escudero (ICMAT-UAM).

Supervisors: Ana Bravo (ICMAT-UAM) and Santiago Encinas (University of Valladolid).

Date: January, 2018.

Beatriz Pascual Escudero. Algebraic varieties are the central objects of study in algebraic geometry. In their simplest terms, an (affine) algebraic variety of dimension *n* over a field *k* is simply the set of solutions of a finite system of polynomials in *n* variables with coefficients in the field *k*. For example, $X = \{(x, y, z) \in C^3: x^2 + z^3 - y^2z^2 = 0\}$ is an algebraic variety (two-dimensional) over the field of complex numbers. Furthermore, it is often required to be irreducible; that is, that it cannot be written as the union of other varieties (otherwise these other varieties would be its *irreducible components*).

Singularities usually constitute an obstacle for the application of many results that are known when no singularities appear, which means that they require special attention with respect to the other points. They are therefore the object of study in different branches of mathematics. From a geometric point of view, the singular points of a variety are those where the dimension of the tangent space is greater than that of the variety itself. From an algebraic point of view, the singular points correspond to multiple roots of polynomials; in commutative algebra, the singular points correspond to non-regular local rings. An algebraic variety is said to be singular if it possesses singular points.

In particular, many results in algebraic geometry have only been demonstrated for the non-singular points of a variety. One possible way of extending these results to singular varieties is to approximate each singular variety X through a non-singular variety X' that differs from the former solely at the singular points, and from which a certain type of function $X' \rightarrow X$ of a singular variety may be defined. This function must respect the structure of the variety, which is what is known as a morphism, and is also required to fulfill certain additional properties. Thus, in order to extend some results to all types of varieties, it would be enough to determine that they hold for any non-singular variety X', and for any variety with which X' may be associated by means of one these morphisms.

The so-called *Problem of Resolution of Singularities* consists of deciding whether for any singular variety X a morphism of this type can be found from a non-singular variety. More particularly, given an algebraic variety defined over some field *k*, a *resolution of singularites of X* is a non-singular variety X' together with a *proper* and *birational* $X' \rightarrow X$ morphism. The term *birational* means that dense open sets of X and X' exist where the morphism is an isomorphism; that is, this morphism is required to be bijective at all the points, except in a relatively small subset. The condition of being *proper* ensures that it is possible to extend many properties of X' to X. It is often also necessary to define an isomorphism specifically outside the singular points of X (which in fact are a relatively small set). A type of morphism that fits this description is what is known as a *blow-up*: a morphism (birational and proper) that transforms the points of a certain closed subset of X, called the *centre of the blow-up*, into a subset of X', known as an *exceptional divisor*. However, the *blow-up* establishes a one-to-one correspondence between the points of X that are not at the centre and those of X' that are not in the exceptional divisor.

An affirmative answer to the problem of resolution of singularities would open the way to extend to the general case many results from algebraic geometry that are only known to hold for non-singular varieties. Moreover, resolution of singularities is employed as a tool in some other proofs, and an affirmative answer would enable results in fields such as motivic integration and positivity to be proven. For example, some Lojasiewicz-type identities are proved by using resolution of singularities.

At present, it is known that a resolution of singularities always exists, provided that the variety X is defined over a field of *characteristic zero* (such as complex or rational fields, etc.). This result is a theorem by the Japanese mathematician Heisuke Hironaka (1964), for which he received a Fields Medal in 1970. Some partial results are known for fields of positive characteristic (thanks to Shreeram S. Abhyankar, Joseph Lipman, Vicent Cossart-Olivier Piltant, Ana Bravo-Orlando E. Villamayor, Angélica Benito-Orlando E. Villamayor and Hiraku Kawanoue-Kenji Matsuki, among others), but the general case remains an open problem.

Hironaka's answer to the problem in characteristic zero is that for any variety a resolution of singularities can be found (which in fact is not unique), defined as a sequence of transformations; specifically, blow-ups in centres that are non-singular closed subsets. However, Hironaka's proof only states that there exists thereby a sequence of blow-ups, without providing any procedure for defining it. Other results have subsequently emerged, some of which are constructive, such as those by Orlando E. Villamayor, Edward Bierstone-Pierre D. Milman, and later others by Santiago Encinas-Orlando E. Villamayor, Santiago Encinas-Herwig Hauser, Jaroslaw Włodarczyk and János Kollár.

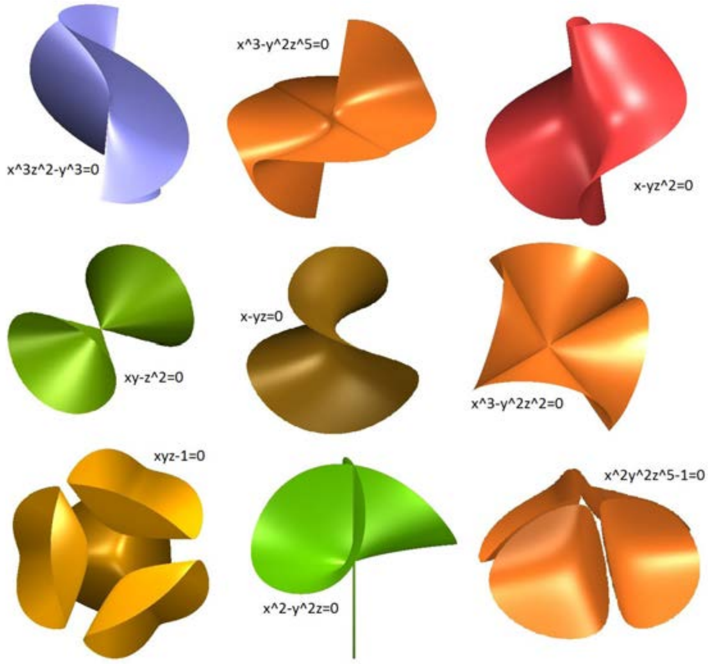
The so-called *constructive (or algorithmic) resolution of singularities* seeks to design an algorithm which, for any variety, unequivocally determines the construction of a birational morphism, given by a sequence of carefully chosen blow-ups. This algorithm must be capable of choosing, for any variety X, a closed subset of X that is the best centre for a blow-up, in accordance with some established criterion aimed at linking together a chain of blow-ups that form a resolution of singularities of X.

For the design of algorithms with these characteristics, *invariants* associated with the points of X are employed; that is, numbers (or sequences of numbers) that do not depend on the representation chosen for the variety (intrinsic to the variety) and which give a measure of the complexity of the points, thereby enabling them to be mutually and absolutely compared (within the same variety or in different varieties, without depending, for instance, on what equations are chosen for describing each point or each variety). To that end, these invariants must be capable of distinguishing between different types of singularity. The study of these invariants is interesting for the design of the algorithm, but they also provide information about the resolution phenomenon (about the relation a variety has with its possible resolution of singularities and about how this process works), which may help to resolve the problem in more general contexts. Some invariants commonly used for this purpose are the Hilbert-Samuel function and the multiplicity.

The *Problem of Resolution of Singularities* provides motivation for defining invariants of the singular points of varieties, as well as being associated with other methods of studying such varieties; for example, from the point of view of algebra, geometry and topology. The study of invariants of singularities is also interesting, for instance, in the classification of varieties.

The *resolution invariants* also constitute one of the main objects of my thesis. In particular, i have studied certain tools known as *arc spaces* from the point of view of constructive resolution of singularities. Arcs have been useful for the understanding of some geometric and topological properties of varieties, as shown in works by Jan Denef-François Loeser, Lawrence Ein, Shihoko Ishii, Mircea Mustaţă, Ana Reguera and Takehiko Yasuda, to name just a few.

Image: Beatriz Pascual



Monomial surfaces.

Arc spaces also arise as tools for the study of singularities of algebraic varieties in the work of the Nobel Prize and Abel Prize laureate John F. Nash. Technically speaking, if we have a variety X defined over a field k, an *arc* in X is a ring homomorphism defined from the ring of the variety to the ring of formal power series in one variable K[[t]], for any field K that contains k. The simplest example of an arc is the case in which K=k, when an arc corresponds with an infinitesimal neighbourhood of a curve contained in the variety X (a formal curve). The set of all the arcs in a variety also has an algebraic structure (scheme) and is known as the *arc space* (although it is not a variety). In a paper that was not published until some years after the fact, John Nash posed the question of whether the irreducible components of the arc space of a variety would contain information about the possible resolutions of the said variety (in terms of its exceptional divisors, at least those divisors that appear in all of them, the so-called *essential* divisors).

In my thesis we have investigated the possible connections between invariants of singularities that arise in terms of the arc space of a variety and the information (the invariants) that are often employed for defining resolution algorithms. In particular, we have concentrated on the *Nash multiplicity sequence*. This is a non-decreasing sequence of positive integers $m_0 \geq m_1 \geq m_2 \geq \cdots$ that is associated with each arc of a variety X. This sequence, defined first of all by Monique Lejeune-Jalabert for hypersurfaces and later generalized by Michel Hickel, could be understood as a certain generalization of the *invariant multiplicity* (which tells us the intensity of a variety at a point), which would be measured along directions of formal curves (given by the arcs).

On the basis of this sequence, in the thesis we define some invariants for the arcs that in a certain way measure the contact each arc has with the set of points of the variety X where the multiplicity is the highest possible. By taking into account all the arcs of a variety that pass through a certain singular point (specifically, a point of maximum multiplicity), these invariants provide us with information about how intricate the arcs passing through this point may be, which ultimately gives us a measure of how the singularity is. In addition, we prove that these invariants are related to others that are frequently used in the case of characteristic zero for the algorithmic resolution. In particular, the most important invariant for the algorithmic resolution (the so-called *Hironaka order*) appears naturally in the arc space given by these new invariants that we define.

These results constitute the first step towards establishing a dictionary between the world of arcs and that of constructive resolution. It is interesting to note that (as we have demonstrated subsequent to the thesis) the definition of our invariants does not actually depend on the fact that the field k is of characteristic zero, which means that these numbers are significant for the variety in terms of arcs, and this occurs with complete generality for any perfect field. On this basis, we think that it could be very useful to discover other numbers that may arise from arcs in a similar way and which may serve as invariants for the study of the resolution problem in fields of any characteristic.

SHE DOES MATHS: Rosana Rodríguez López

Image: Rosana Rodríguez

Rosana Rodríguez is an expert in differential equations and diffuse mathematics.

Rosana Rodríguez López is a teacher and researcher at the University of Santiago de Compostela, where in 2005 she completed her PhD with a thesis entitled "Periodic solutions for nonlinear differential equations." A member of the former Department of Mathematical Analysis and of the current Department of Statistics, Mathematical Analysis and Optimization, she is the Vice Dean of the Faculty of Mathematics and Degree Coordinator for Mathematics where she also teaches, a task she combines with a fruitful research career. Prior to holding this post, she was awarded a *Formación para Personal Investigador* (FPI – Research Training) grant as well as teaching in secondary schools, an experience she decribes as "enriching".

Research Fields:

Differential Equations, Fuzzy Mathematics.

Javier Fuertes. Rosana Rodríguez López achieved worldwide recognition from the mathematical community in 2005, when together with Juan José Nieto, her thesis supervisor, she published the article "Contractive Mapping Theorems in Partially Ordered Sets and Applications to Ordinary Differential Equations" in the journal "Order". Since then she has authored some 70 papers on mathematical analysis in which she has developed techniques for finding more precise solutions to problems in which uncertainty is the key.

Her research fields fall into three main categories, the first of which, functional differential equations, concerns the study of systems where previous studies are decisive for their current configuration. One good example consists of population models "in which future generations depend directly on the characteristics of those that have gone before." Her second area of research concerns fractional differential equations, which are useful in processes where some particular memory is invloved, such as in the physics of resistance and diffusion. Although based on classical concepts, these equations have enjoyed a revived interest in recent decades, since, as she explains, "according to the type of derivative employed, they enable their singularities to be found." Finally, Rosana also conducts research on analaysis techniques in fuzzy mathematics, based on the Lofti Zadeh theory of fuzzy logic. This type of analysis is used when the complexity of the process in question is very high and no precise mathematical models are available, or when the process is subjective or not strictly defined. An example of its applications would be in decision-making and the mathematical study of linguistic processes.

"Developing techniques to find more precise solutions to problems in which uncertainty is the key"

In addition to her work in research and in teaching coordination, Rosana also devotes time to outreach, particularly in the area in and around Santiago de Compostela. She is also currently supervising three PhD theses, the first on fractional calculus and the other two on differential models. We may imagine the smile on her face when she remarks: "Encouraging young people to tackle and embrace what fires them with enthusiasm is also what encourages you to persevere."

SCIENTIFIC REVIEW: Long term dynamics for the restricted N -body problem with mean motion resonances and crossing singularities

Original title: "Long term dynamics for the restricted N -body problem with mean motion resonances and crossing singularities".
Authors: Stefano Marò (ICMAT) y Giovanni Federico Gronchi (Università di Pisa).
Source: *SIAM Journal on applied dynamical systems,* 17(2), 1786–1815.
Date of (online) publication: 19 June 2018.
Link: https://epubs.siam.org/doi/10.1137/17M1155703

Celestial mechanics is concerned with the study of the motions of celestial bodies under the action of gravitational forces. Despite possessing a clear physico-astronomical connotation, this discipline is closely related to mathematics. Firstly, celestial mechanics employs the formalism and the ideas of mathematics, for which furthermore is an inexhaustible source of inspiration.

For example, Gauss for the first time used with success the least squares method to determine the orbit of the first asteroid (Ceres), and the restricted three-body problem led Poincaré to establish the foundations of modern chaos theory.

Differential equations are used to describe the motion of planets and asteroids; specifically, differential equations with a Hamiltonian structure. The study of these systems is a classical subject in applied mathematics, and celestial mechanics makes great use of these techniques. For example, thanks to perturbation theory it is possible to find simple systems that approximate well to what one observes in the sky. Furthermore, the results of the numerical integrations of the equations can be interpreted in the light of the theory of Hamiltonian systems.

One of the greatest problems in current celestial mechanics concerns the study of the dynamics of asteroids. Most of these small celestial bodies in the solar system follow elliptical orbits in the region between the orbits of Mars and Jupiter, which is known as the asteroid main belt. However, the perturbations arising from the gravitational influence of these two major planets, especially Jupiter, have caused a considerable number of asteroids to leave the main asteroid belt and enter into orbits quite close to the Earth. These asteroids are known as Near Earth Asteroids (NEA).

From a mathematical point of view, the motion of an asteroid is modelled as a restricted N-body problem; the dynamics of the asteroid is given by the gravitational influence produced by the Sun and the Planets. The corresponding system of differential equations falls within the framework of near-integrable non-autonomous Hamiltonian systems with three degrees of freedom, the position of the planets being a known function of time. The influence of the planets constitutes a small perturbation of the dynamics due to the gravitational influence of the Sun, and this latter system (Sun-asteroid) behaves in accordance with the Kepler model: the asteroid describes an elliptical path with the Sun at one of the foci, and the position on the ellipse is given by the Kepler equation.

The position of the asteroid and the planets in a heliocentric system are described by six coordinates divided into two groups: five geometric coordinates that describe the trajectories at each moment, and a sixth coordinate that determines the position throughout the trajectory. In Kepler's problem, the five geometric variables remain constant, since the elliptical orbit does not change. However, if the influence of the planets is also taken into account, these variables cease to be constant, although they evolve very slowly (in periods of time much longer than the evolution of the sixth coordinate).

For this reason, the geometric variables are known as "slow" and the sixth as "fast". The Hamiltonian structure of the problem then consists of two parts; one main part that corresponds to the Kepler problem and depending only on the slow variables, and a remainder that is proportional to a small parameter corresponding to the perturbations of the planets and containing dependencies of the fast coordinates of the asteroid and the planets.

From an intuitive point of view, this description by coordinates shows that the asteroid moves along an ellipse that deforms very slowly. This is a mechanism by means of which an asteroid may leave the main belt and approach the Earth as its trajectory changes.

It is then interesting to study the evolution of the asteroid's trajectory, disregarding its position throughout that trajectory. The equations describing the slow coordinates are deduced by the Hamiltonian perturbation theory. The idea is to eliminate the fast coordinates (of both the asteroid and the planets) from the expression of the Hamiltonian by means of an average. On a highly intuitive level, we contemplate a system in which the asteroid and the planets are scattered all along their trajectories.

Formally, this is achieved by a canonical change of variable. The result is a new Hamiltonian consisting of three parts: the main part, then a first remainder proportional to the small parameter, and a second remainder proportional to the square of the small parameter. The effect of changing the variable is to *move* the dependency of the fast variable from the first remainder to the second. The Hamiltonian tought is found by ignoring the second remainder and is often referred to as the normal form. The execution of this plan is very complicated if a proportionality exists between the period of revolution of the asteroid and a planet. In this case, *mean motion resonance* is said to exist. However, it is possible to obtain a normal form that is known as the resonant normal form.

In either case, with or without resonances, theorems exist which state that the evolution according to the normal form is a good approximation to the real evolution. For more information on this question, one should consult the following reference: Morbidelli A., Modern celestial mechanics, Taylor & Francis, 2002.

These theorems are valid providing that there are no singularities corresponding to intersections between the asteroid and a planet. In that case, the normal Hamiltonian form is not differentiable, so the corresponding vector field is not continuous and it is not possible to arrive at a solution in the classical sense. Nevertheless, asteroids that pass close to the Earth often present intersections between their trajectories, which in principle makes the study of the dynamics by the normal form impossible.

In recent years, a possible solution has been advanced in the case without resonance (Gronchi, G.F., Tardioli, C: Secular evolution of the orbit distance in the double average restricted three-body problem with crossing singularities, Discrete Contin. Dyn. Syst. Ser. B 18 (2013), 1323-1344). This shows that it is possible to define a generalized solution that *passes through* the singularities. These solutions are not regular with respect to time at the moment of the intersection, but it can be shown that they are Lipschitz-continuous. Moreover, numerical experiments suggest that they may constitute a good approximation of the real evolution of the slow variables.

Stefano Marò (ICMAT) and Giovanni Federico Gronchi (Università di Pisa) have recently extended this theory to chaos with resonance, taking into account the substantial differences that it presents. For example, if one assumes a resonance with Jupiter, it is necessary to distinguish whether the orbit of the asteroid intersects the orbit of Jupiter itself or that of another planet (e.g. the Earth). Likewise in this case, numerical experiments also suggest that the generalized solution is a good approximation of the real evolution of the slow variables. A formal demonstration of these phenomena poses a challenge for future work.

SCIENTIFIC REVIEW: Brauer correspondent blocks with one simple module

Original title: "Brauer correspondent blocks with one simple module".
Authors: Carolina Vallejo (ICMAT), Gabriel Navarro (Universitat de València) and Pam Huu Tiep (Rutgers University).
Source: *Journal of Algebra*.
Date: In press.

Summary. Finite groups model the symmetries that occur in nature. There are two classical ways of studying them; either by their actions on sets (the Theory of Permutation Groups) or by their actions on vector spaces (Representation Theory). By studying the action of a group G on a vector space V, what we do is study the representation of G as a sub-group of GL(V) that arises. The so-called Character Theory studies the trace of the representation. Characters were first defined by Frobenius in 1896. One year later, Burnside writes in the preface of his "Theory of groups of finite order" that in dealing purely with the theory of groups, no more concrete mode of representation should be used than is absolutely necessary. However, in 1911, in the preface of the second edition of his book he recognizes how useful the Representation Theory has been for the the Theory of groups of finite order and writes "the reason given in the original preface for omitting any account of it no longer holds good". Not even Burnside was totally convinced from the beginning.

In this paper entitled *Brauer correspondent blocks with one simple module*, the researchers show how the p-local structure of a finite group G determines and is determined by the character theory of the principal p-block of G.

More information:

In this article, "Brauer correspondent blocks with one simple module", we show that the p-local structure of a finite group G determines and is determined by the character theory of the principal p-block of G for any odd prime p. The case where p=2 is treated in [NV17] and [ST18].

The irreducible characters of a finite group G contain most of the relevant information about the actions of G on complex vector spaces. A character χ of G is a class function χ: $G \to \mathbb{C}$ that codifies this information. For a prime number p that divides the order of G, the characters of G are partitioned into Brauer p-blocks, the principal block being the only one containing the character 1_G that corresponds to the trivial action of G on $\mathbb{C}$. The p-blocks appear when considering at the same time the ordinary and modular characters of G (the actions of G over complex vector spaces and spaces of positive characteristic p).

In the context of finite groups, the word «p-local» is related to the structure of the p-subgroups of G and its normalizers, the most paradigmatic case being that in which the p-subgroups are Sylow subgroups. A Sylow p-subgroup P is a subgroup of G of order p^a, where p is the greatest power of p that divides the order of G, and its normalizer $N_G(P)$ is the set of elements $g \in G$ such that $g^{-1}Pg = P$ (namely those elements of G that act on P).

In the 1960s, Richard Brauer related the p-blocks of G with p-blocks of normalizers of p-subgroups of G. This is the well-known Brauer correspondence. An essential idea in Representation and Character Theory is to characterize the local structure of a group G by looking only at its global character theory. For odd primes p, we prove that the action of $N_G(P)$ on P by conjugation only generates inner automorphisms if, and only if, 1_G is the only character of degree not divisible by p and p-rational contained in the principal p-block of G. The condition on $N_G(P)$ is equivalent to saying that $N_G(P)$ is p-nilpotent or that $N_G(P)$ decomposes as a direct product of P and a group of order not divisible by p. With regard to the conditions on the characters of the principal p-block, on the one hand, the condition on the degree of the characters is related to the McKay conjecture and, on the other hand, the condition of p-rationality has to do with a conjecture by Gabriel Navarro that refines the McKay conjecture by taking into account the values of the characters.

A character χ *of* G is associated with the action of G on a complex vector space. The degree of χ is the dimension of such a vector space. The McKay conjecture (for p = 2 it is now a theorem by [IMN07] and [MS16]) predicts that the number of characters of degree not divisible by p of G and of $N_G(P)$ is the same. John L. Alperin remarked that there must exist a bijection between these sets, which preserves the partitions of characters into p-blocks. These two conjectures are fundamental open problems in Group Representation Theory. Going one step further, Gabriel Navarro conjectured [Nav04] that there must also exist a bijection preserving the fields of values of the characters calculated over the p-adics (the field of values of χ over $\mathbb{Q}_p$ is obtained by adjoining to $\mathbb{Q}_p$ the values of χ). In particular, there must exist a bijection that preserves the number of p-rational characters (whatever they are, since I am not defining them) on both sides. This conjecture is attracting a great deal of interest from the community over recent years, see for example [Ruh17], [BN18], [NSV18]. It is worth mentioning that the Navarro conjecture implies our main result in "Brauer correspondent blocks with one simple module" so that our work supports this amazing conjecture.

One example of the surprising nature of the global-local conjectures in Representation Theory is that of the Monster group. The Monster group M is one of the 26 sporadic simple groups (and one of the key ingredients in the Monstruos Moonshine conjecture). It gets its name from its enormous size, since it is a group with $2^{46} \cdot 3^{20} \cdot 5^{9} \cdot 7^{6} \cdot 11^{2} \cdot 13^{3} \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 \approx 8 \cdot 10^{53}$ elements. Its character theory is very complex; nevertheless, according to the above-mentioned global-local conjectures, in order to study the irreducible actions of G on complex odd-dimensional vector spaces (or at least, certain aspects of them), it suffices to study the same type of actions of one of its Sylow 2-subgroups P (since in this case $N_M(P) = P$). Such a group has $2^{46} \cdot$ so it is tiny compared to the Monster.

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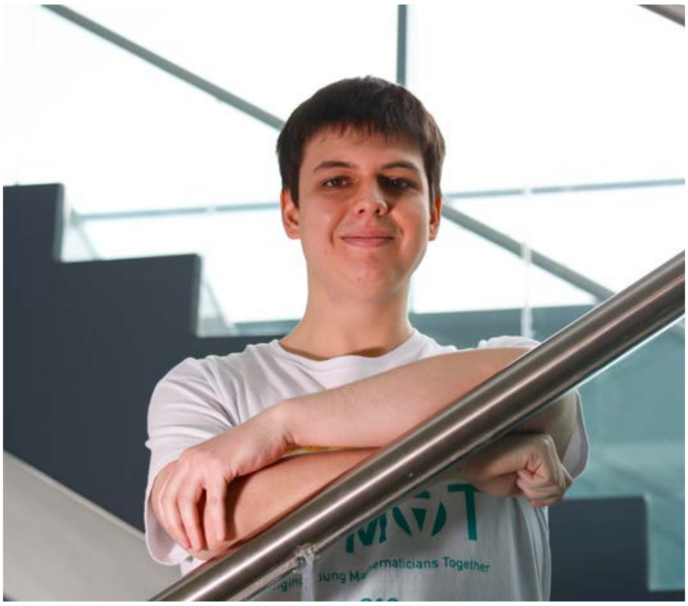
PROFILE: David Alfaya, predoctoral student at the ICMAT

Answers enclosed in space

Image: BBVA Foundation



David Alfaya (ICMAT) was one of the organizers of the BYMAT congress, which took place last May at the ICMAT.

David Alfaya (Madrid, 1990) has had a lifelong interest in science: physics, chemistry, robotics, informatics... Perhaps that is why he likes most of all mathematics, the language and nexus common to all the sciences. Choosing only one path when you are interested in many is not an easy task, so Alfaya opted for two degree courses, one in Mathematics and the other in Engineering Informatics, for the second of which he was awarded the End of Course First Prize in his year. During this period he obtained two JAE Mathematics grants for introduction to research at the ICMAT. In the first he worked on the Ramsey Theory and in the second he was able to work alongside ICMAT researcher Tomás Gómez. At the end of that summer he did his Master in Mathematics and Applications at the UAM, where he completed his end of course assignment under the supervision of Gómez. At present, thanks to a predoctoral grant from La Caixa, he is engaged in research on moduli spaces, again with the same researcher. In parallel, he has also worked on informatics research, jointly with the Information Retrieval Group at the UAM, as well as on search and recommendation algorithms similar to those employed by Amazon and Youtube.

Javier Fuertes. David Alfaya is putting the finishes touches to his PhD thesis entitled "Automorphism Group of the Moduli Space of Parabolic Vector Bundles Over a Curve", which he is doing under the supervision of Tomás Gómez. A theoretical physicist and mathematical research at the ICMAT, Gómez has infused Alfaya with his passion for the relation between certain geometric objects (moduli spaces) and physics. "Moduli spaces are spaces that provide a geometric interpretation of classification problems of objects, such as the solutions to a differential equation. The idea is to find a geometric object that represents all of these solutions and which, on moving through the space it defines, shows how these solutions change." In this case, the purpose is to classify fibre bundles that "are applied in fields of physics, such as the Nigel Hitchen autodual equations and equations belonging to particle physics".

Alfaya's mathematical adventure began with the *Estímulo Matemático* (Estalmat) program in Madrid, in which he first participated at the age of ten. This was followed by the Mathematical Olympiad at both national and international level. His participation in this competition took him to Romania, Vietnam, Brazil, and Portugal, and the experience he acquired there, like the training for an olympic athlete, launched him on what turned out to be his professional career.

"As a part of my preparation, I learned a lot about many kinds of interesting and enjoyable mathematics that were very different from what is usually studied in class, and much closer to what a real mathematician or researcher actually does. That's where my future became clear," he explains in an affable tone, evoking the good times he had preparing for the competitions and stimulating his curiousity. Despite the apparent clarity about his future, Alfaya has had to narrow down his broad range of scientific interests. He also took part in the Physics and Chemistry Olympiads, winning a gold medal in the national version of the former. "The topics in the informatics part of quantum information have also interested me very much. I tried to find mathematical applications to quantum fields, but now the mathematics I'm engaged is applied to particle physics," he says with a chuckle, "and honestly I'd like to know more about the subject, because at the moment my knowledge in this area is limited."

While not wishing to dwell on the subject, he speculates on the complexity underlying the physics part of his research work.

Simultaneously he has been involved in informatics research, working with the UAM Information Retrieval Group, which is devoted to search and recommendation algorithms similar to those used by Amazon and Youtube. Indeed, Improvements can be made to these algorithms thanks to the analysis of a geometric shape, which appears when the results of the search are represented in a space. Alfaya also eventually completed a Master in Research and Innovation in Communication Technology. Where on earth did he find the time? "In summer," he says, "when I had a little more free time".

There is no doubt that this young scientist is inspired more by a sense of vocation than a series of goals to be achieved. As he says: "I first started by getting involved in my present branch of research, moduli spaces, with Tomás; then in research into geometry and information. The two fields intersect."

Having tried both sides of the coin, David is inclined towards pure research. "It's a question of what you like," of feeling at home untying the Gordian knot of abstract thought. "You use geometry, and when you get to a halt, that's where algebra begins. And when algebra stops you can start on analysis and the differential equations that lie beneath. When the differential equations come to an end, you get back to geometry. What I like best is the abstraction involved in regarding problems as *manifolds*. Moduli theory is the representation of all of this type geometric thought, which after all is abstract."

He is tempted in the future to recover his former interests, but he knows that they would have to be tackled in parallel with the brilliant career in mathematics he began to forge for himself ten years ago. "I'll carry on doing research," he says. "I'm absolutely sure about that." These words are uttered with a conviction that leaves no room for doubt.

Diego Córdoba awarded a grant of 1.8 million euros for the study of equations in fluid dynamics

Image: ICMAT

Diego Córdoba has been awarded with an ERC Advanced Grant, the most important scientific prize in Europe.

Diego Córdoba, ICMAT Research Professor and Principle Researcher of the Severo Ochoa programme at the Centre, has obtained an Advanced Grant from the European Research Centre (ERC), the highest distinction on the continent for a scientist. His project, entitled "Non-local dynamics in incompressible fluids", has been awarded funding to the tune of 1.8 million euros over the next five years for the development of new methods for analyzing partial differential equations.

"In particular, I'm interested in equations for modelling the motion of fluids," said Córdoba. These expressions are obtained by applying Newton's laws of conservation (force is equal to mass multiplied by acceleration) to a fluid volume. In this project, Córdoba and his team will study the equations of incompressible fluid flow (that is, when pressure is exerted on one side of an incompressible fluid, it expands on the other, just like a liquid). As Córdoba explains: "We start from regular conditions of departure of the variables, and want to find out how they evolve over time."

These Advanced Grants are awarded to currently active researchers who, according to the ERC, are "outstanding leaders in terms of the originality and the importance of their scientific contributions." Since the inception of the programme in 2007, only six of these grants in the field of mathematics have been awarded to researchers belonging to Spanish institutions. Nine ERC Advanced Grants have been extended to researchers in mathematics this year throughout Europe, and Diego Córdoba is the only Spaniard among those recipients. He had previously obtained an ERC project Starting Grant the first time they were awarded in 2007.

The first national outreach network for mathematics is launched

More than 50 popularizers and communicators of mathematics met in Zaragoza on May 10th and 11th as part of the "Technologies in mathematical outreach" day sessions, which constituted the founding event of the new Red de Divulgación Matemática (DiMa – Mathematical Outreach Network). This was announced in a manifesto in which some 60 professionals advocated recognition for the dissemination of their science, since as stated in the document, "never has the need for the understanding of mathematics and the capacity for using it at work and in everyday life been greater than they are now." "It seems that after years of being the ugly duckling of the sciences, mathematics has at last become fashionable, and the interest it generates has not stopped growing," say the signatories of the manifesto. "That's why we believe that now is the time to bring about a qualitative change in the dissemination of mathematics in our country."

The idea is to unite the efforts of all those involved in mathematical outreach. Among those affiliated to the network are prestigious communicators such as Claudi Alsina (University of Barcelona), Raúl Ibáñez (University of the Basque Country UPV-EHU), Clara Grima (University of Seville), Antonio Pérez and Marta Macho-Stadler (UPV-EHU). "Everone dedicated to outreach is necessary for taking up the challenges that face us," says Edith Padrón, DiMa coordinator and a Professor at the University of la Laguna. The main aim of the network is the sharing of experiences, material, learning and thoughts on the subject of communication involving the mathematical community as a whole, together with help from the relevant institutions.

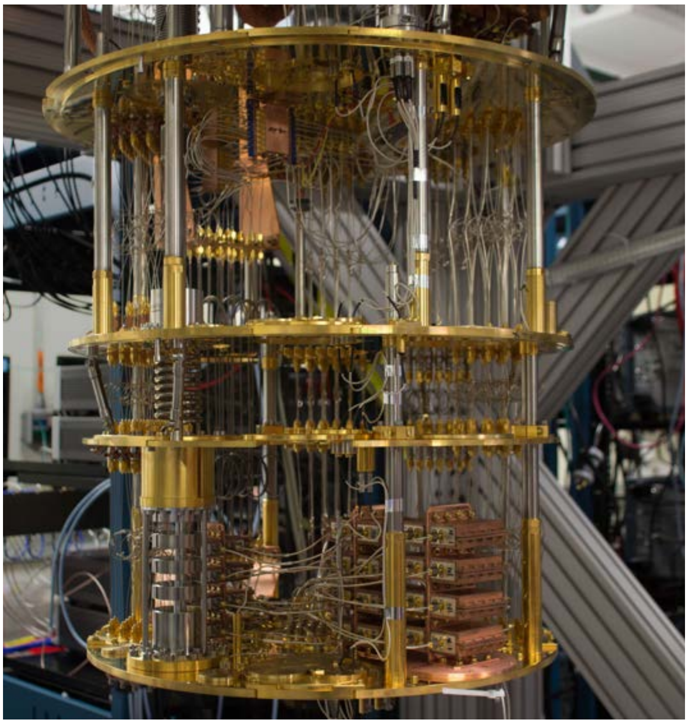
Future activity of the network includes a meeting of mathematical outreach communicators like the one held in Zaragoza, as well as the creation of an outreach school. Network profiles can be found on Twitter, Facebook and a website.

Image: DiMa

DiMa was born in the 'Tecnologías en la divulgación matemáticas' conference, held in Zaragoza.

The origins of the DiMa network reside in a previous meeting held in January, 2007, at the ICMAT (Madrid), which was attended by mathematical communicators and outreach workers with the purpose of analyzing the need for a network of this kind. The ICMAT has at its disposal an outreach infrastructure that is exceptional in Spain, since it has its own Mathematical Culture Unit, the only one in the field of mathematics recognized by the Spanish Foundation for Science and Technology (FECYT) network of Scientific Culture Units (UCC). The centre drew up a strategic communication plan that was launched in 2012 with the granting of the Severo Ochoa project and coordinated by ICMAT personnel contracted expressly for that purpose.

Design of a new fail-safe cryptographic method for quantum computers



Ignacio Luengo (ICMAT-UCM) has designed a new encrypted system against quantum computers.

Research into quantum computers is proceeding apace. While the construction of these machines on a large scale is still uncertain, more and more scientists regard it as a distinct possibility within the next 20 years. Were this to happen, the cryptographic systems employed today to protect e-Business on the web would cease to be effective. It is for this reason that the National Institute of Standards and Technology (NIST) in the USA is giving priority to the creation of new and secure algorithms for both conventional and quantum computers (known as post-quantum systems).

To this end, in 2016 the NIST issued an invitation for public tenders to identify, select and standardize systems of this type. Among the 83 submissions from 17 different countries accepted in 2017, 53 were chosen, one of which was submitted by Ignacio Luengo, a professor at the Complutense University of Madrid and a member of the Institute of Mathematical Sciences (ICMAT). As this researcher explains: "Our encryption method promises to be secure against attacks on future quantum computers, as well as being fast and effective in encryption, decryption and digital signature processes."

On April 11th and 13th of this year, Luengo presented his method at a congress held in Fort Lauderdale (Florida, USA). The first stage of evaluation will be concluded at the end of 2018, and the second will last for five years. The standardization process is expected to be completed in 2015, and several "winners" are also expected, at least one for each technology included in the tender.

The urgency to get the process under way as soon as possible is because of the need not only to ensure the security of data in the future, but also data at present. Organizations possessing a quantum computer within the next twenty years would be able to decode all the traffic on the Internet in the period before the introduction of post-quantum cryptography. Some of this data will have no value, but other data such as medical records or state and industrial secrets require confidentiality and protection for much longer periods of time. The transition process will be complex and it will be necessary to prioritize data encryption, providing protection for the most sensitive data as soon as possible.

Carolina Vallejo, postdoctoral researcher at the ICMAT, and Álvaro del Pino, former PhD student at the Institute, each awarded a 2018 Vicent Caselles Prize by the RSME and the BBVA Foundation

Image: ICMAT

Carolina Vallejo.

Álvaro del Pino.

Image: Álvaro del Pino

Every year since 2015, the RSME (Royal Spanish Mathematical Society) and the BBVA Foundation have awarded the Vicent Caselles Prizes for Mathematical Research to young researchers under 30 who are in the first stages of their scientific careers. This prize is a recognition of the "work, creativity, originality and results" of the young prize-winners.

One of the award-winners this year is Carolina Vallejo, a postdoctoral researcher at the ICMAT whose work is based on finite group representation theory; specifically, the study of the relations between the characters of a group and its subgroups. The jury stressed "her research on the so-called McKay conjecture, which has enabled her to resolve some cases and have been essential for results by other authors." Among the other prize-winners is Álvaro del Pino, currently a postdoctoral researcher at the University of Utrecht (the Netherlands) and a former ICMAT PhD student of differential topology under the supervision of Francisco Presas (ICMAT-CSIC). He was awarded this prize for his "significant results on the classification of Engel structures and the theory of symplectic foliations."

In addition to Vallejo and Del Pino, others distinguished with the award are David Beltrán, postdoctoral researcher at the BCAM (Basque Center for Applied Matehmatics); David Gómez Castro, researcher at the Complutense University of Madrid Institute of Multidisciplinary Mathematics and member of the Pontificia Comillas University Department of Applied Mathematics; David González Álvaro, postdoctoral researcher at the University of Fribourg (Switzerland), and Vanesa Guerrero, visiting professor at the Carlos III University Department of Statistics in Madrid.

Former ICMAT doctoral student and current collaborator, Javier Gómez, receives the SeMA "Antonio Valle" Prize for young researchers

The 21st SeMA (Spanish Society of Applied Mathematics) "Antonio Valle" Prize for young researchers was this year awarded to Javier Gómez Serrano, assistant professor at Princeton University (USA) and a member of the Charles Fefferman ICMAT Laboratory. This young researcher received the prize during the celebration of the Jacques-Louis Lions Hispano Francesa School on Numerical Simulation in Physics and Engineering that was held in Gran Canaria between June 25th and 29th.

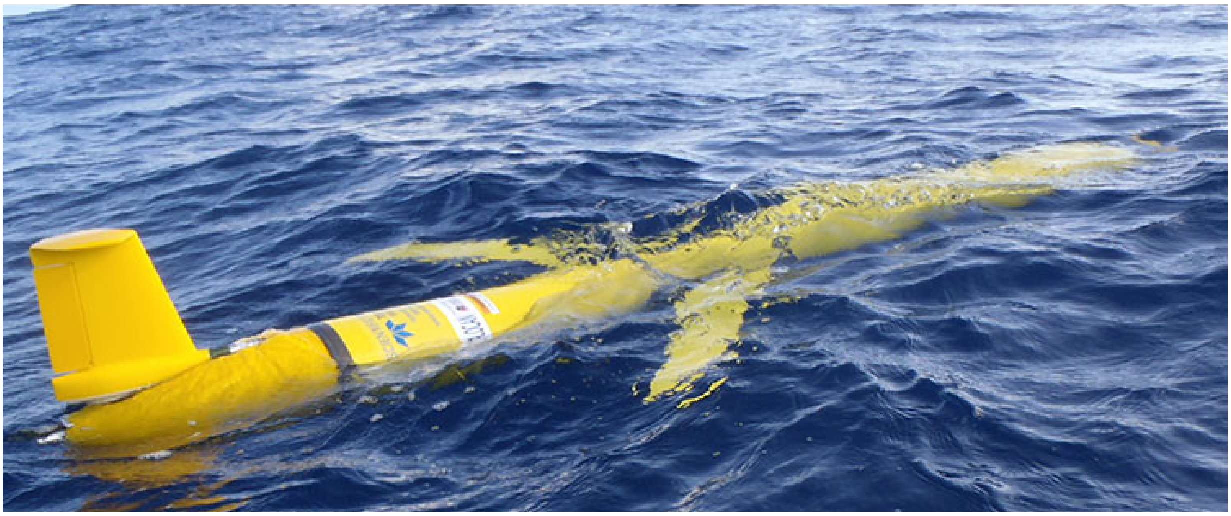
The contributions that earned him the award were remarked by the SeMa in a communiqué. Gómez Serrano completed his PhD at the ICMAT under the supervision of Diego Córdoba, Principal Researcher of the Severo Ochoa programme and member of the ICMAT. His work is focused on the analysis of partial differential equations; in particular, in fluid dynamics. In 2017, he was also distinguished with the Vicent Caselles Prize, which is awarded by the RSME and the BBVA Foundation.

Image: ICMAT

Javier Gómez.

A mathematical method enables an underwater drone to reach unprecedented speeds

Image: rucool.marine.rutgers.edu/gliders/silbo/



Silbo glider during the Challenger Glider Mission.

A mathematical method devised by researchers at the *Instituto de Ciencias Matemáticas* (ICMAT) has enabled Slocum underwater drone gliders, vehicles used to explore the ocean depths, to reach unprecedented speeds with low battery consumption. These speeds were achieved in a mission that crossed the north Atlantic between April, 2016, and March, 2017, as part of the Challenger Glider Missions in which mathematicians, oceanographers and engineers participated. According to ICMAT researcher Ana María Mancho: "On the basis of the ever-changing ocean currents, this mathematical approach sketches a Lagrangian structure that identifies dynamic barriers." The details and conclusions of the mission were published in March this year in the journal Scientific Reports.

In order to steer these vehicles, scientists communicate with them in real time when they emerge at the surface at periodically programmed times, although it is essential to take into account the ocean currents that effect the performance of the drones by slowing them down or speeding them up according to their direction. Víctor García-Garrido, co-author of the study, says that "mathematics enables an optimum route to be determined in accordance with the dynamic turbulence of the ocean by analyzing these currents."

For Mancho, "this experience has shown that it is possible to find optimum trajectories in a turbulent ocean by means of the identification of robust structures, thereby increasing the possibilities of oceanic exploration." Furthermore, this data has enabled the reliability of the available representations of ocean currents to be verified. This methodology could be applied the the study of the ocean and the atmosphere in different contexts. To date, temperature maps obtained by satellite measurements had been used to design trajectories for these gliders.

Slocum gliders are autonomous submarine vehicles whose use in oceanography is becoming increasingly extensive for the exploration of the ocean depths at a very low cost. Their energy consumption is practically zero, but only for communication and measurement purposes and control of their angle of dive. Mancho goes on to say that, "they are able to cover great distances; they function with a propulsion mechanism that uses changes in floatability, and they enable data to be collected in areas of the ocean that are difficult to access, such as those located beneath tropical cyclones and the ice caps in polar regions."

Eduardo Sáenz de Cabezón in Mathematics at the Residencia

Image: ICMAT

Eduardo Sáenz de Cabezón gave the "El número que los ordenadores nunca podrán calcular" talk in the 'Matemáticas at the Residence' series.

The attempt to reach immensely large numbers has occupied mathematics for centuries. More than setting new records, these numbers running into the millions are necessary for describing our cosmos. For example, more than 2,000 years ago Archimedes invented the numbers required for counting the number of grains of sand that fit into the universe. Now, the passwords that protect transactions online are shielded by calculations that even the fastest computers would take years to carry out. While the security of these algorithms appears to be threatened by quantum computation, the calculation capability of which will eventually be far greater than that we enjoy today, could another type of number be found that could not be computed even by quantum computers?

The mathematician, scientific disseminator and humourist Eduardo Sáenz de Cabezón spoke about this and other questions in his talk "The number that no computer could ever calculate", which he gave in March this year as part of the series "*Matemáticas en la Residencia*" and organized by the Institute of Mathematical Sciences (ICMAT) in collaboration with the Vicepresidency of Organization and Scientific Culture of the Spanish National Research Council (CSIC), together with the *Residencia de Estudiantes*. The talk was introduced by Carles Mesa, broadcaster and director of the *Radio Nacional de España* (RNE) programme Gente Despierta (People Awake).

"It was Turing who questioned whether there were limits to computation, even before computers existed", says Sáenz de Cabezón, who is also a professor at the University of Logroño. "And indeed there are; numbers of such magnitude exist that no computer could ever calculate." And this "is not only important for cybernetic security, but also implies the existence of algorithms so complex that they cannot be resolved, at least, not in a reasonable amount of time."

Mathematics enables the prediction of new properties of graphene

Since Andre Geim (Sochi, 1958) and Konstantin Novoselov (Nizhny Tagil, 1974) demonstrated the stability of graphene at room temperature – an achievement for which they received the Nobel Prize for Physics in 2010 – "by means of simplified models these, physicists have demonstrated experimentally that graphane possesses incredible electronic, thermal, optical and mechanical properties, although we still do not know why," says Charles Fefferman, the winner of a Field Medals and the Wolf Prize, as well as the director of an ICMAT Laboratory, during one of his recent visits to the centre. He goes on to say that: "This is the point at which mathematics comes into play, which may be able to explain this phenomenon theoretically." The study of graphene equations is one more example of the long and prolific relation between physics and mathematics. In this case, the most important contribution according to Fefferman "is the development of equations that best represent the real nature of graphene. From the theoretical point of view, one of the most interesting features of this material would be to determine how electrons move around in it. This would allow us to manipulate the charges in graphene, which in turn would have applications to the construction of faster quantum computers and to the development of superconductors, among many other things."

The model of the electron in graphene, which is what Fefferman is working on, attempts to describe how these negatively charged particles behave in the lattice of carbon atoms. As this researcher, who is currently at Princeton University (USA), points out: "Although it is assumed that the electrons do not interact with each other as they move through the graphene, which is a simplification, the model is more realistic than those currently used by physicists".

Image: ICMAT

Charles Fefferman studies graphene's equations.

The outreach series "It's a risky life!" draws to an end

Image: ICMAT

David Ríos and Aisoy robot have been the main characters in 'It's a risky life!' audiovisual project.

The outreach series "It's a risky life!", produced by the AXA Research Fund and the Institute of Mathematical Sciences (ICMAT), has come to an end. The eighth and final episode came out in January of this year. It was entitled "Adversaries" and featured the holder of the AXA-ICMAT Chair in Adversarial Risk Analysis, David Ríos, who explained in a genial tone how mathematics can help when it comes to predicting the behaviour of the so-called intelligent adversaries, such as terrorists and hackers. It is from this type of issue that adversarial risk analysis has arisen, a field that enables these dangerous situations to be anticipated or forestalled and the best decisions in such circumstances to be taken.

Throughout the eight episodes, these videos have presented vital mathematical concepts regarding risk and their influence on society. The series has addressed subjects such as security, decision-making, uncertainty and aversion to risk from a mathematical perspective. After each episode a challenge was issued with the aim of making the audience think about the notion of risk in everyday situations. The person who has answered the most questions correctly will receive a prize of an Aisoy programmable social robot.

Thematic trimester: "L^2-invariants and their analogues in positive characteristic"

The third ICMAT thematic trimester in 2018 was devoted to [L^2-invariants], a recently developed area in algebraic geometry, which has its origins in the work of the English mathematician, Michael Atiyah, Fields Medallist and Abel Prize winner. This research field has proved to be very useful in the study of important problems such as the Baum-Connes conjecture and the Hanna Neumann conjecture, as well as being a source of ideas and tools for tackling some of the most pertinent open problems in various disciplines. Furthermore, the study of L^2-invariants is associated with topology, geometry, global analysis, operator theory, ring theory, group theory and the K theory.

The first gathering in this thematic trimester consisted of a research school, which was held at the Institute between February 26th and March 9th this year. This initial event was followed by various advanced courses and group discussions, and finally a conference on the latest developments in the field that brought the trimester to a close in June.

Image: ICMAT

The Introductory school was the first activity of the L^2- invariants thematic program.

Thematic trimester: "Real harmonic analysis and its applications to partial differential equations and geometric measure theory"

Image: ICMAT

Participants on the "workshop on real harmonic analysis and its applications to partial differential equations and geometric measure theory: on the occasion of the 60th birthday of Steve Hofmann".

Between the months of May and June this year, the Institute of Mathematical Sciences (ICMAT) organized a programme on real harmonic analysis and its applications to partial differential equations and geometric measure theory. Ten seminars were held in which the latest advances in the field were presented; three mini-courses on research devoted to the training of young researchers and given by Steve Hofmann (the University of Missouri, USA), Tatiana Toro (University of Washington, USA) and Xavier Tolsa (ICREA and the Autonomous University of Barcelona), and a conference at which mathematicians of world renown presented their latest achievements. All together, some one hundred researchers from more than 20 countries came to the ICMAT to take part in these activities.

The main topic of the programme was the field of mathematics known as harmonic analysis and its applications to the study of partial differential equations in specific contexts. Among other things, harmonic analysis studies waves such as sound waves, their diffusion and their relation with the medium.

AGENDA

ICMAT scientific activities

Research program on Moduli Spaces
Date: 15 September - 15 December 2018.
Place: ICMAT (Madrid).

ICMAT outreach activity

Science Week 2018

- **Math scavenger hunt.** Ángela Capel and Jesús Ocáriz.
  Date: 5 November 2018.
  Place: C.C Pablo Iglesias (Alcobendas, Madrid).
- **Conference: "Cuento todo el día, con o sin compañía".** Luis Rández.
  Date: 6 November 2018.
  Place: C.C Pablo Iglesias (Alcobendas, Madrid).

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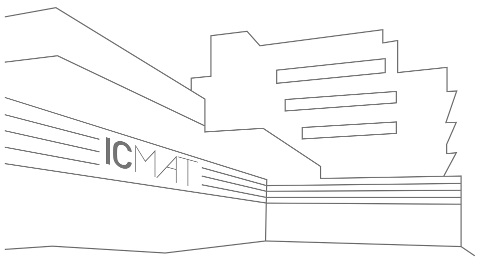

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