FACTORIZATION AND DOMINATION OF POSITIVE BANACH-SAKS OPERATORS

JULIO FLORES AND PEDRO TRADACETE

Abstract. It is proved that every positive Banach-Saks operator $T : E \to F$ between Banach lattices E and F factors through a Banach lattice with the Banach-Saks property, provided that F has order continuous norm. By means of an example we show that this order continuity condition cannot be removed. In addition, some domination results, in Dodds-Fremlin sense, are obtained for the class of Banach-Saks operators.

1. Introduction

Factorization properties of operators between Banach spaces have been widely studied. It is well known that every weakly compact operator between Banach spaces factors through a reflexive Banach space [6]. In contrast, M. Talagrand showed that, in general, a weakly compact operator between Banach lattices cannot be expected to factor through a reflexive Banach lattice [20]. However, C. D. Aliprantis and O. Burkinshaw proved that this can be done under quite general assumptions [1].

Regarding the class of Banach-Saks operators between Banach spaces, Beauzamy provided in [5] an analogous factorization result. Namely, every Banach-Saks operator factors through a Banach space with the Banach-Saks property. In this note we want to consider the analogous question in the Banach lattice context. In particular, we will prove that every positive Banach-Saks operator from a Banach lattice to an order continuous Banach lattice, factors through a Banach lattice with the Banach-Saks property (see Theorem 2.1). In addition, we show that Talagrand's example [20] can be used to prove that the order continuity hypothesis cannot be removed (see Example 1).

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In the second part of this note we give a domination result for the class of Banach-Saks operators (Corollary 3.3) which improves the results previously obtained in [12]. Recall that the domination problem for a class $\mathcal{C}$ of operators acting between Banach lattices is stated as follows:

Problem 1. *Let $0 \leq R \leq T : E \to F$ be two positive operators. Assume that T belongs to the class $\mathcal{C}$. Which conditions on E and F do ensure that R belongs to $\mathcal{C}$?*

For positive endomorphisms on a Banach lattice the *power problem* is closely related:

Problem 2. *Let $0 \leq R \leq T : E \to E$, and $T \in \mathcal{C}$. Which is the smallest $n \in \mathbb{N}$ such that $R^n \in \mathcal{C}$?*

Corollary 3.2 provides an answer to this question for the class of Banach-Saks operators.

Power and domination problems have been studied for other classes of operators in [3], [4], [7], [14], [21], and more recently in [11] and [15].

We refer to the books [16], [17] and [2] for unexplained terms and notation.

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2. Factorization of operators through Banach lattices with the Banach-Saks property

Recall that an operator between Banach spaces $T : X \to Y$ is *Banach-Saks* if every bounded sequence (x_n) in X has a subsequence such that (Tx_{n_k}) is Cesàro convergent, that is, the sequence of arithmetic means $(\frac{1}{N}\sum_{k=1}^{N} T(x_{n_k}))_N$ is convergent in the norm of Y. A Banach space is said to have the *Banach-Saks property* if the identity operator is Banach-Saks. We say that a Banach space X has the *weak Banach-Saks property* if every weakly null sequence in X has a Cesàro convergent subsequence.

For convenience, we will say that a subset S of a Banach space X is a *Banach-Saks set* if for every sequence (x_n) in S, there exists a subsequence (x_{n_k}) that is Cesàro convergent. Clearly an operator $T : X \to Y$ is Banach-Saks if and only if $T(B_X)$ is a Banach-Saks set, where B_X is the closed unit ball of X.

The main result of this note is the following:

Theorem 2.1. *Let E and F be Banach lattices and $T : E \to F$ a positive Banach-Saks operator. If F is order continuous, then there exist a Banach* *lattice H with the Banach-Saks property, and operators $T_1 : E \to H$, $T_2 : H \to F$, such that the following factorization diagram holds:*

$$\begin{tikzcd} E \arrow[rr, "T"] \arrow[dr, "T_1"'] & & F \\ & H \arrow[ur, "T_2"'] & \end{tikzcd}$$

Before the proof, we need to collect some definitions and facts. Recall that an order continuous Banach lattice E with a weak unit can be considered as an (in general not closed) order ideal of $L_1(\Omega, \Sigma, \mu)$ for certain probability space (Ω, Σ, μ), such that the natural inclusion $E \hookrightarrow L_1(\Omega, \Sigma, \mu)$ is continuous with norm smaller than one [17, Prop. 1.b.14]. Recall also that in an order continuous Banach lattice every ideal is complemented by a positive projection [17, Prop. 1.b].

Let E be a Banach function space with order continuous norm defined over a finite measure space (Ω, Σ, μ). Recall that a bounded subset $A \subset E$ is *equi-integrable* if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $\|f\chi_B\|_E < \varepsilon$ for every $B \in \Sigma$ with $\mu(B) < \delta$ and every $f \in A$. The following result is known (see [10, Lemma 3.3] for a proof).

Lemma 2.2. *Let E be a Banach lattice with order continuous norm and a weak unit, and hence representable as an order ideal in $L_1(\Omega, \Sigma, \mu)$ for some probability space (Ω, Σ, μ). A norm bounded sequence (g_n) in E is convergent if and only if it is equi-integrable and $\| \, \|_1$-convergent.*

Proof of Theorem 2.1. First of all, we claim that the solid hull of $T(B_E)$ is a Banach-Saks set.

Indeed, since T is Banach-Saks, $T(B_E)$ is a Banach-Saks set. Let W denote the solid hull of $T(B_E)$. Take an arbitrary sequence (z_k) in W. This sequence satisfies $|z_k| \leq |Tx_k|$ for certain $x_k \in B_E$. For every k, take $r_k = |x_k|$ in B_E, and then

$$|z_k| \leq |Tx_k| \leq Tr_k.$$

Since T is Banach-Saks, there exist $g \in F$ and some subsequence (k_s) such that

$$\frac{1}{m} \sum_{s=1}^{m} Tr_{k_s} \to g$$

in the norm of F. In fact, by [8] we can assume that the same property holds for every subsequence of (k_s).

Take M, the closed ideal of F generated by $x = g + \sum_{s=1}^{\infty} \frac{z_{k_s}}{2^s}$. According to the remarks above, denote $i : M \hookrightarrow L_1(\Omega, \Sigma, \mu)$ the continuous inclusion of M as an order continuous Banach lattice with weak unit into some $L_1(\Omega, \Sigma, \mu)$, and let $P : F \to M$ be a positive projection onto M.

Clearly, the composition

$$iPT : E \to L_1(\Omega, \Sigma, \mu)$$

is a Banach-Saks operator. In particular, $iPT(B_E)$ is relatively weakly compact and so is its convex solid hull, as $L_1(\Omega, \Sigma, \mu)$ is a band in its bidual [2, Theorem 4.39]. Hence, there exists $f \in L_1(\Omega, \Sigma, \mu)$ such that $i(z_{k_s}) \to f$ in the weak topology of $L_1(\Omega, \Sigma, \mu)$. Since $L_1(\Omega, \Sigma, \mu)$ has the weak Banach-Saks property [19], passing to a further subsequence we can assume that

$$\frac{1}{m} \sum_{s=1}^{m} i(z_{k_s}) \to f$$

in the norm of $L_1(\Omega, \Sigma, \mu)$.

Notice that

$$\left| \frac{1}{m} \sum_{s=1}^{m} z_{k_s} \right| \leq \frac{1}{m} \sum_{s=1}^{m} Tr_{k_s},$$

and $\frac{1}{m} \sum_{s=1}^{m} Tr_{k_s} \to g$ in the norm of F. It follows that $\left(\frac{1}{m} \sum_{s=1}^{m} z_{k_s} \right)_m$ is equi-integrable in F, and therefore convergent in F by the previous lines and Lemma 2.2. Thus, W is a Banach-Saks set as claimed.

Consider now F_0, the completion of the space $\{z \in F : \exists \lambda < \infty,\ z \in \lambda W\}$ under the norm induced by the Minkowski functional of W. Since W is solid and convex, the space F_0 is in fact a Banach lattice. Hence, the space $(F_0, F)_{\theta,p}$ $(0 < \theta < 1,\ 1 < p < \infty)$, obtained by Lions-Peetre interpolation of F_0 and F, is a Banach lattice too [17, 2.g]. Moreover, by [5, Thm. 2], $(F_0, F)_{\theta,p}$ has the Banach-Saks property.

Finally, since $T(B_E) \subset W$, the operator $T : E \to F_0$ is bounded. Thus, by the interpolation theorem [17, Prop. 2.g.15], T is bounded from E to $(F_0, F)_{\theta,p}$. Let $T_1 : E \to (F_0, F)_{\theta,p}$ denote this operator. Since $W \subset B_F$, we also have that the inclusion $i : (F_0, F)_{\theta,p} \hookrightarrow F$ is bounded. Therefore, we have the factorization

$$\begin{tikzcd}
E \arrow[rr, "T"] \arrow[dr, "T_1"'] & & F \\
& (F_0, F)_{\theta,p} \arrow[ur, hook, "i"'] &
\end{tikzcd}$$

Take $H = (F_0, F)_{\theta,p}$, and $T_2 = i$ to conclude the proof. $\square$

Corollary 2.3. *Let E be a Banach lattice. If $0 \leq T : E \to E$ is Banach-Saks, then T^2 factors through a Banach lattice with the Banach-Saks property.*

Proof. Since c_0 does not have the Banach-Saks property, T cannot be an isomorphism on any subspace of E isomorphic to c_0. Hence, by [13, Thm. I.2], there exist an order continuous Banach lattice F, and positive operators R, S such that

$$\begin{tikzcd} E \arrow[rr, "T"] \arrow[dr, "R"'] & & E \\ & F \arrow[ur, "S"'] & \end{tikzcd}$$

Therefore, since F is order continuous, Theorem 2.1 yields that $RT : E \to F$ factors through a Banach lattice H with the Banach-Saks property. Hence, $T^2 = SRT$ also factors through H as claimed. $\square$

Note that, in general, every Banach-Saks operator between Banach spaces factors through a Banach space with the Banach-Saks property [5, Thm. 1]. However, if the operator acts between Banach lattices it is not true in general that the space obtained in such a factorization has to be a lattice. To see this we will benefit from the well-known example provided by Talagrand [20] of a positive weakly compact operator between Banach lattices which fails to factor through any reflexive Banach lattice. Since Banach-Saks property implies reflexivity it suffices to prove that Talagrand's operator is in fact Banach-Saks. Thus, Theorem 2.1 (and Corollary 2.3) turns out to be optimal in a sense.

Let us briefly recall for the reader's convenience the construction of Talagrand's operator. First, let

$$L = \{h : \mathbb{N} \cup \{\infty\} \to \{0,1\};\ \exists\, p \leq i_1 < \ldots < i_p;\ h(i) = 0 \text{ for } i \neq i_1, \ldots, i_p\}.$$

Then $L \subset C(\mathbb{N} \cup \{\infty\})$ is weakly compact. For every $l \geq 1$ consider the map

$$\theta_l : L^l \to C((\mathbb{N} \cup \{\infty\})^l),$$

defined as $\theta_l(h_1, \ldots, h_l)(n_1, \ldots, n_l) = 1$ if the number of indexes i for which $h_i(n_i) = 1$ is even, and $\theta_l(h_1, \ldots, h_l)(n_1, \ldots, n_l) = 0$ otherwise. Notice that since θ_l is continuous for the topology of point-wise convergence in $C((\mathbb{N} \cup \{\infty\})^l)$, the set $K_l = \theta_l(L^l)$ is weakly compact.

Call M the Alexandroff compactification of the discrete sum of the sets $(\mathbb{N} \cup \{\infty\})^l$. Each K_l can be considered as a subset of $C(M)$ by extending the functions of K_l to zero outside $(\mathbb{N} \cup \{\infty\})^l$. Let $K = \bigcup_l K_l$. By construction,

K consists of $\{0,1\}$-valued functions, so K is contained in the positive cone of $C(M)$.

Lemma 2.4. *The closed convex hull of K, $\overline{co}K$, is a Banach-Saks set.*

Proof. Indeed, take (y_n) arbitrarily in $\overline{co}K$. We want to show that there is a subsequence of (y_n) whose arithmetic means are convergent. For each $n \in \mathbb{N}$ write

$$y_n = \sum_{j=1}^{\infty} \lambda_{n,j} w_{n,j},$$

where $\sum_{j=1}^{\infty} \lambda_{n,j} = 1$, $\lambda_{n,j} \geq 0$, and $w_{n,j}$ belongs to $W_j = \overline{co}K_j$, the closed convex hull of K_j.

Passing to a subsequence of (y_n), we can assume that for all $j \in \mathbb{N}$, there exists λ_j such that $\lambda_{n,j} \xrightarrow{n} \lambda_j$, with $\sum_{j=1}^{\infty} \lambda_j = 1$. Let

$$y'_n = \sum_{j=1}^{\infty} \lambda_j w_{n,j},$$

and

$$e_n = y_n - y'_n.$$

Since K_j is weakly compact, so is W_j; hence, passing to a further subsequence we can assume that for each $j \in \mathbb{N}$ there is some $z_j \in W_j$ such that $w_{n,j} \xrightarrow{n} z_j$ weakly. Note that for each $j \in \mathbb{N}$, W_j is weakly compact in $C((\mathbb{N} \cup \{\infty\})^j)$, which is isomorphic to c_0. Since c_0 has the weak Banach-Saks property [9], we obtain that W_j is a Banach-Saks set. Hence, using [8] and a diagonal process, we can extract a subsequence (n_i) such that for each $j \in \mathbb{N}$ there exists $f_j : \mathbb{N} \to \mathbb{R}$ satisfying

$$\left\| \sum_{i=1}^{k} w_{n_i,j} - k z_j \right\| \leq f_j(k)$$

and $\frac{f_j(k)}{k} \to 0$ when $k \to \infty$. Since the W_j are disjointly supported on M we get

$$\left\| \sum_{i=1}^{k} y'_{n_i} - k \sum_{j=1}^{\infty} \lambda_j z_j \right\| = \left\| \sum_{j=1}^{\infty} \lambda_j \Big(\sum_{i=1}^{k} w_{n_i,j} - k z_j \Big) \right\| \leq \max_j \lambda_j f_j(k),$$

which implies

$$\frac{1}{k} \sum_{i=1}^{k} y'_{n_i} \xrightarrow{k} \sum_{j=1}^{\infty} \lambda_j z_j$$

in the norm of $C(M)$. Hence, (y'_{n_i}) has convergent arithmetic means (and also every subsequence of it).

A gliding hump argument yields that (e_n) has a subsequence equivalent to the unit vector basis of c_0. Indeed, taking an appropriate subsequence we can assume that $\lambda_{n,j} \to \lambda_j$ fast enough, so that the following construction can be carried out. First, set $n_1 = 1$ and let j_1 be such that

$$\Big\| \sum_{j=j_1}^{\infty} (\lambda_{n_1,j} - \lambda_j) w_{n_1,j} \Big\| < \frac{1}{2}.$$

Next, take n_2 such that

$$\Big\| \sum_{j=1}^{j_1} (\lambda_{n_2,j} - \lambda_j) w_{n_2,j} \Big\| < \frac{1}{2^3},$$

and then choose j_2 such that

$$\Big\| \sum_{j=j_2}^{\infty} (\lambda_{n_2,j} - \lambda_j) w_{n_2,j} \Big\| < \frac{1}{2^3}.$$

In this way, we construct inductively a pair of sequences (n_k) and (j_k) such that

$$\sum_{k=1}^{\infty} \Big\| e_{n_k} - \sum_{j=j_{k-1}}^{j_k - 1} (\lambda_{n_k,j} - \lambda_j) w_{n_k,j} \Big\| \leq 1.$$

Thus, (e_{n_k}) is equivalent to $\big(\sum_{j=j_{k-1}}^{j_k} (\lambda_{n_k,j} - \lambda_j) w_{n_k,j} \big)_k$ which is a disjoint sequence in $C(M)$ equivalent to the unit vector basis of c_0.

Finally, note that every subsequence of the unit vector basis of c_0 has convergent arithmetic means. Therefore, both (e_{n_k}) and (y'_{n_k}) have subsequences with the same property. This implies that the same is true for some subsequence of (y_n) and the proof is finished. $\square$

Example 1. *There exists a positive operator $U : \ell_1 \to C[0,1]$ which is Banach-Saks but it fails to factor through a Banach lattice with the Banach-Saks property.*

Proof. Note that K can be seen as a subset of $C[0,1]$ by taking a positive embedding of $C(M)$ into $C([0,1])$, such that its image is complemented. Take (x_n) a dense sequence in K and consider the operator

$$\begin{array}{cccc} U : & \ell_1 & \longrightarrow & C([0,1]) \\ & (a_n)_{n=1}^{\infty} & \longmapsto & \sum_{n=1}^{\infty} a_n x_n. \end{array}$$

Clearly U is positive. Moreover, $U(B_{\ell_1}) = co(K)$ is a Banach-Saks set by Lemma 2.4, and therefore the operator U is Banach-Saks.

By [20, Thm. A] the operator $U : \ell_1 \to C([0,1])$ does not factor through any reflexive Banach lattice. Since every space with the Banach-Saks property is reflexive, the proof is finished. $\square$

This shows that the hypothesis on Theorem 2.1 are necessary. Moreover, if we consider the operator $\widetilde{U} : \ell_1 \oplus C([0,1]) \to \ell_1 \oplus C([0,1])$, given by $\widetilde{U}(x,y) = (0, U(x))$, then one notices that Corollary 2.3 cannot be improved.

3. Domination of Banach-Saks operators

In this section we look at the problem of domination for Banach-Saks operators on Banach lattices. Recall that an operator $T : E \to X$, from a Banach lattice E to a Banach space X is *order weakly compact* if T maps order intervals to weakly precompact sets [2, p. 318]. Observe that given $0 \leq R_1 \leq T_1 : E_1 \to E_2$ and $0 \leq R_2 \leq T_2 : E_2 \to E_3$, with T_1 Banach-Saks and T_2 order weakly compact, the proof of Theorem 2.1 can be adapted to obtain the factorization

$$\begin{tikzcd}
E_1 \arrow[rr, shift left, "T_1"] \arrow[rr, dashed, shift right, "R_1"'] \arrow[dr, shift left, "P_1"] \arrow[dr, dashed, shift right, "Q_1"'] & & E_2 \arrow[rr, shift left, "T_2"] \arrow[rr, dashed, shift right, "R_2"'] & & E_3 \\
 & & H \arrow[urr, shift left, "P_2"] \arrow[urr, dashed, shift right, "Q_2"'] & &
\end{tikzcd}$$

where H is a Banach lattice with the Banach-Saks property, $0 \leq Q_1 \leq P_1$ and $0 \leq Q_2 \leq P_2$. From here a domination result for Banach-Saks operators is easily obtained. However, we provide an alternative proof of this fact which does not depend on interpolation. This is the content of the following result which improves some previous work in [12].

Theorem 3.1. *Let E_1, E_2 and E_3 be a Banach lattices and $0 \leq R_i \leq T_i : E_i \to E_{i+1}$ be positive operators for $i = 1, 2$. If T_1 is a Banach-Saks operator, and T_2 is order weakly compact then the composition $R_2 R_1$ is a Banach-Saks operator.*

Proof. Since T_2 is order weakly compact, by [13, Thm. I.2] we have the factorization:

$$\begin{tikzcd}
E_1 \arrow[r, shift left, "T_1"] \arrow[r, dashed, shift right, "R_1"'] & E_2 \arrow[rr, shift left, "T_2"] \arrow[rr, dashed, shift right, "R_2"'] \arrow[dr, "\phi"'] & & E_3 \\
 & & F \arrow[ur, shift left, "P"] \arrow[ur, dashed, shift right, "Q"'] &
\end{tikzcd}$$

where F is an order continuous Banach lattice, and $0 \leq Q \leq P$.

Take an arbitrary sequence (x_n) in B_{E_1} and the closed ideal generated by $(|\phi T_1(x_n)|)$ in $F \hookrightarrow L_1(\Omega, \Sigma, \mu)$ as above.

Since T_1 is Banach-Saks and

$$|\phi R_1 x_{n_k}| \leq \phi T_1 |x_{n_k}|,$$

there exists a subsequence (n_k) such that the arithmetic means $\frac{1}{m}\sum\limits_{k=1}^{m}\phi T_1|x_{n_k}|$ converge to some $x \in F$. Then $\left(c_m = \frac{1}{m}\sum\limits_{k=1}^{m}\phi R_1 x_{n_k}\right)_m$ is an equi-integrable sequence in F (and the same is true for any subsequence of (n_k) by [18]).

Since ϕT_1 is weakly compact, Gantmacher's theorem implies in particular that its adjoint $(\phi T_1)^*$ is order weakly compact, so we get a factorization for ϕT_1 and ϕR_1 through a Banach lattice G, such that both G and G^* are order continuous [13, Prop. I.4 and Thm. I.6]:

$$\begin{tikzcd}
E_1 \arrow[rr, "T_1"] \arrow[rr, dashed, "R_1"'] \arrow[dr, "U"] \arrow[dr, dashed, "V"'] & & E_2 \arrow[rr, "T_2"] \arrow[rr, dashed, "R_2"'] \arrow[dr, "\phi"'] & & E_3 \\
& G \arrow[rr, "\psi"'] & & F \arrow[ur, "P"] \arrow[ur, dashed, "Q"'] &
\end{tikzcd}$$

By passing to some subsequence, [2, Theorem 4.25] yields that $(\phi R_1 x_{n_k})$ is weakly Cauchy, hence weakly convergent in $L_1(\Omega, \Sigma, \mu)$. Now, by [19], $(\phi R_1 x_{n_k})$ has a subsequence whose arithmetic means converge in the norm of $L_1(\Omega, \Sigma, \mu)$ to some function $f \in L_1(\Omega, \Sigma, \mu)$. However, since

$$\left|\frac{1}{m}\sum_{k=1}^{m}\phi R_1 x_{n_k}\right| \leq \frac{1}{m}\sum_{k=1}^{m}\phi T_1|x_{n_k}|,$$

and

$$\frac{1}{m}\sum_{k=1}^{m}\phi T_1|x_{n_k}| \to x$$

for some $x \in F$, we must have $|f| \leq x$, which implies that $f \in F$. Therefore the sequence of arithmetic means, (c_m), must be convergent in the norm of F (see Lemma 2.2). This implies that ϕR_1 and consequently $R_2 R_1$ are Banach-Saks operators. This finishes the proof. $\square$

Corollary 3.2. *Let E be a Banach lattice and $0 \leq R \leq T : E \to E$ be positive operators. If T is Banach-Saks, then R^2 is also Banach-Saks.*

Proof. Since T is Banach-Saks, it is also weakly compact [18], and in particular order weakly compact. Theorem 3.1 yields the result. $\square$

Note that in [12, Ex. 2.9] it was shown that there exist operators

$$0 \leq R \leq T : \ell_1 \to \ell_\infty$$

such that T is Banach-Saks, but R is not. This shows that Corollary 3.2 is sharp; indeed, consider the operators $0 \leq \widetilde{R} \leq \widetilde{T} : \ell_1 \oplus \ell_\infty \to \ell_1 \oplus \ell_\infty$ defined by

$$\widetilde{R} = \begin{pmatrix} 0 & 0 \\ R & 0 \end{pmatrix} \qquad \widetilde{T} = \begin{pmatrix} 0 & 0 \\ T & 0 \end{pmatrix}.$$

Clearly $\widetilde{T}$ is Banach-Saks, but $\widetilde{R}$ is not. Notice that $\widetilde{R}^2 = 0$.

We also have the following improvement to [12, Thm. 1.1].

Corollary 3.3. *Let E and F be Banach lattices, such that F is order continuous. If $0 \leq R \leq T : E \to F$, with T Banach-Saks, then R is also a Banach-Saks operator.*

Proof. Use Theorem 3.1 and the fact that order intervals in an order continuous Banach lattice are weakly compact [17, p. 28]. □

The following question remains open: Can order continuity on F be replaced with order continuity on E^* in Corollary 3.3?

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Department of Applied Mathematics, Escet, Universidad Rey Juan Carlos, 28933, Móstoles, Madrid, Spain.

E-mail address: julio.flores@urjc.es

Departamento de Análisis Matemático, Facultad de CC. Matemáticas, Universidad Complutense de Madrid, 28040, Madrid, Spain.

E-mail address: tradacete@mat.ucm.es