PhD THESIS DEFENSE

SYLVESTER RANK FUNCTIONS, EPIC DIVISION RINGS, AND THE STRONG ATIYAH CONJECTURE FOR LOCALLY INDICABLE GROUPS

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Abstract

Throughout the thesis we consider questions related to embeddings of noncommutative domains into division rings.

On the one hand, we treat the problem of existence of such embeddings for group rings $K[G]$ where $K$ is a subfield of the field of complex numbers $\mathbb{C}$ and $G$ is a locally indicable group (for instance, a torsion-free one-relator group). These groups comprise a subfamily of left orderable groups, and hence this is a particular instance of the Malcev embedding problem.

In this sense, the strong Atiyah conjecture for these group rings, principal and original motivation to the thesis, proposes a candidate to be a division ring in which $K[G]$ embeds, namely, the division closure of $K[G]$ inside the classical quotient ring $U(G)$ of the group von Neumann algebra $\mathcal{N}(G)$. In the main result (joint with A. Jaikin-Zapirain) we prove that the strong Atiyah conjecture holds in this setting and that, moreover, the resulting division ring can be uniquely identified through a universal property. The associated methods and results allow us to prove a posteriori other related conjectures, such as a version of Lück’s approximation conjecture for virtually locally indicable groups.

On the other hand, we deal with the notion of universality of a division ring. For a ring $R$, a universal division ring of fractions is a division ring that contains and is generated by $R$ as a division ring, and in which we can invert “the most” matrices possible over $R$. In this regard, (pseudo)-Sylvester domains are rings $R$ admitting a universal division ring of fractions in which every matrix becomes invertible unless there is an “obvious” obstruction.

In a joint work with F. Henneke, we prove that crossed products of the form $E \ast G$, where $E$ is a division ring and $G$ is free-by-$\{\text{infinite cyclic}\}$, are always pseudo-Sylvester domains, and we explore the more general situation of crossed products $\mathfrak{F} \ast \mathbb{Z}$ of a fir $\mathfrak{F}$ and the ring of integers $\mathbb{Z}$.

Along the thesis, the theory of Sylvester rank functions provides a unifying language and a tool to address the problems considered. We further analyze the space of Sylvester rank functions that can be defined on certain families of rings, including Dedekind domains and a subfamily of skew Laurent polynomial rings with coefficients in a division ring (joint with A. Jaikin-Zapirain).