This thesis centers on the study of two different problems of partial differential equations arising from geophysics and fluid mechanics: the surface quasi-geostrophic equation and the so called, Incompressible Slice Model. The surface quasi-geostrophic equation is a two dimensional nonlocal partial differential equation of geophysical importance, describing the evolution of a surface buoyancy in a rapidly rotating, stratified potential vorticity fluid.

In the first part of the talk, we will present some global regularity results for its dissipative analogue in the critical regime for the two dimensional sphere. After that, we will introduce the Incompressible Slice Model dealing with oceanic and atmospheric fluid motions taking place in a vertical slice domain R2, with smooth boundary. The ISM can be understood as a toy model for the full 3D Euler-Boussinesq equations. We will study the solution properties of the Incompressible Slice Model: characterizing a class of equilibrium solutions, establishing the local existence of solutions and providing a blow-up criterion.