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L^2-INVARIANTS seminar

SYLVESTER MATRIX RANK FUNCTIONS ON CROSSED PRODUCT ALGEBRAS

VENUE: Sala Naranja, ICMAT (Campus de Cantoblanco, Madrid)

DATE: Tuesday 24 April 2018 - 15:00 h

SPEAKER: Joan Claramunt (UAB, Spain)

ABSTRACT:

Let X be the Cantor set together with a homemomorphism $T : X \to X$, and μ an ergodic T-invariant probability measure on X. For an arbitrary field K, T gives an action of $\mathbb{Z}$ on the set of locally constant functions $X \to K$, denoted by $C_K(X)$, so one can consider the algebraic crossed product $\mathcal{A} := C_K(Z) \rtimes_T \mathbb{Z}$.

We explain a technique of approximating $\mathcal{A}$ by subalgebras $\mathcal{A}_n$ embeddable into infinite matrix products $\mathcal{R}_n = \prod_k^{(n)} M_k(K)$. It turns out that in $\mathcal{R}_n$ we have a natural rank function rk_n compatible with μ, in the sense that $\mu(U) = \mathrm{rk}_n(\chi_U)$ for every clopen $U \subseteq X$, being $\chi_U \in \mathcal{A}_n$ the characteristic function of U.

When glued together, $\mathfrak{R} := \overline{\varinjlim_n \mathcal{R}_n}^{\mathrm{rk}}$ is a regular rank ring, where $\overline{\cdot}^{\mathrm{rk}}$ denotes the rank completion of the inductive limit with respect to the rank function $\mathrm{rk} = \lim_n \mathrm{rk}_n$. The main result is that $\mathcal{A}$ embeds in $\mathfrak{R}$, so $\mathcal{A}$ inherits a natural rank function $\mathrm{rk}_{\mathcal{A}}$ from $\mathfrak{R}$, which is in fact a Sylvester matrix rank function, unique with respect to the property $\mu(U) = \mathrm{rk}_{\mathcal{A}}(\chi_U)$ for every clopen $U \subseteq X$. Moreover, if μ is ergodic, then $\mathrm{rk}_{\mathcal{A}}$ is extremal.

Finally, we compute the rank completion of $\mathcal{A}$, giving $\overline{\mathcal{A}}^{\mathrm{rk}_{\mathcal{A}}} \cong \mathcal{M}_K$ where $\mathcal{M}_K$ is the so-called von Neumann continuous factor, which is the rank completion of the inductive limit $\varinjlim_n M_{2^n}(K)$ with respect its unique rank function. This generalizes a result of Elek [3] in the case $H = \mathbb{Z}$.

More info: https://www.icmat.es/events/seminar/group-theory
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