VENUE: Módulo 17, Facultad de Ciencias, UAM

SPEAKER: Diego Chicharro Gordo (UCM)

ORGANIZER: UAM - ICMAT

LECTURES:

- **Session 1:** Tuesday, 19 March 2019, 17:00 - 18:30 // Aula 520
  Review of algebraic number theory, motivation, definition of the Artin map, and statement of the Artin reciprocity law

- **Session 2:** Tuesday, 26 March 2019, 16:45 - 18:15 // Aula 420
  Analytic methods, the first fundamental inequality

- **Session 3:** Wednesday, 27 March 2019, 16:45 - 18:15 // Aula 420
  Cohomological methods, the second fundamental inequality, and proof of the theorem

ABSTRACT: The quadratic reciprocity law says, in the language of algebraic number theory, that the splitting behaviour of the primes in the imaginary quadratic field \( \mathbb{Q}(\sqrt{D}) \) depends only on the prime modulo \( D \), if \( D \) is chosen appropriately; for instance, a prime \( p \) splits completely in \( \mathbb{Q}(\sqrt{-1}) = \mathbb{Q}(\sqrt{-4}) \) if and only if \( p - 1 \) is multiple of \( 4 \). Hilbert, in his ninth problem, asks for a generalization valid for any extension of number fields \( L/K \). The goal of this course is to understand the statement of the Artin reciprocity law, which solves the case when \( L/K \) is an abelian extension, and to see the main ideas involved in the original proof. We thus start reviewing the basic theory of number fields, defining the Artin map, and formulating the reciprocity law. The next step is to prove the first fundamental inequality, and for that end analytic methods, in particular the Dedekind zeta function and \( L \)-series, are introduced, which we use to study the density of primes in number fields. We then develop briefly Tate cohomology to sketch the proof of the second fundamental inequality. Finally, using the Artin lemma we prove the reciprocity theorem. We also illustrate the usefulness of the theory proving interesting consequences such as the infinitude of primes congruent to 1 modulo a positive integer \( m \), the Kronecker-Weber theorem, which says that any abelian extension if a cyclotomic subextension, and that congruence conditions does not suffice for a nonabelian extension \( L/K \).

Bibliography: