

Study seminar on Sylvester rank functions and L^2 -Betti numbers

Fridays 10:30, ICMAT Aula Gris 2, starting from October 1th, 2021.

Sylvester rank functions are numerical invariants of matrices over an associative ring. They appear naturally in several fields of Mathematics including Group Theory, Operator Algebras, Ring Theory, Topology, and Non-commutative Geometry. They were introduced first by Malcolmson [22] and studied further by Schofield [25].

The seminar is intended to be an introduction to the topic of the Sylvester rank functions with the aim of understanding the L^2 -Betti numbers from an algebraic point of view. We plan also to consider several related topics: mod- p L^2 -Betti numbers, division R -rings and other applications.

The seminar is suitable for master and doctoral students interested in Group Theory, Ring Theory, Topology or Operator Algebras. All participants are invited to give a talk on a topic of their choice from the list below.

Preliminary plan of the seminar:

The list of references is indicative and not complete. The order of the topics can be changed later.

- (1) Introduction to Sylvester rank functions: definition, examples and main properties. [6, Chapter 5], [18, Chapters 1 and 2]
- (2) Epic division R -rings and Malcolmson's theorem. [18, Chapter 3], [3]
- (3) Sylvester rank function on Von Neumann regular rings and $*$ -regular rings. [18, Sections 1.3 and 4.1], [5, Sections 16, 17 and 18], [6, Sections 3 and 5]
- (4) The von Neumann group ring $\mathcal{N}(G)$ and the ring of affiliated operators $\mathcal{U}(G)$. [14, Chapter 1], [24, Chapter 2]
- (5) Representations of groups in \mathbb{C}^* -algebras with trace and associated Sylvester rank functions. The definition of L^2 -Betti numbers. [26, Section 1], [14, Section 2]
- (6) L^2 -Betti numbers of CW -complexes. [14, Sections 3 and 4]
- (7) The Lück approximation. [6, Section 10], [14, Section 5], [19]
- (8) The Atiyah conjecture. [23, Chapter 2], [17]
- (9) The analog of L^2 -Betti number for pro- p groups. [12]
- (10) Mod- p L^2 -Betti numbers. [8]

Additional topics:

- (11) Extended Sylvester rank function. [16]
- (12) Schofield's Theorem on extensions of Sylvester rank function to localizations. [25, Chapter 7], [16, Section 8]
- (13) Sylvester rank function for amenable extensions. [13]
- (14) Sylvester domains and the Atiyah conjecture for free groups. [7], [21]
- (15) The L^2 -polygon and the Thurston polygon.
- (16) The vanishing of the first L^2 -Betti number for an RFRS group and virtually fibrings. [1], [15], [8]
- (17) The Baumslag conjecture on free \mathbb{Q} -groups and constructions of parafree groups. [9], [11]
- (18) The Hanna Neumann conjecture and the Dicks-Ventura conjecture. [2]
- (19) Universal division R -rings and maximal Sylvester rank functions. [8]

- (20) The Kaplansky and and Malcev Conjectures. [6, Section 13], [10]
 (21) The Lück approximation in positive characteristic. [4], [7]

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