Flows of Geometric Structures

Andrés Julián Moreno Ospina (IMECC - Unicamp)

Questions of holonomy reduction to a subgroup H of $\mathrm{SO}(n)$ translate into the existence of parallel tensors. These problems are notoriously hard to solve, however, the geometric flows allow to reach a geometric structure with the "best" geometry, in our case, those with the holonomy group in H. Based on [3], we will study geometric flows of H-structures:

Talk 1 We are going to make a short review about fiber bundles, H-structures, connections and torsion on Riemannian manifolds. Next, we explain how to study infinitesimal deformations of H-structures and we derive the evolution equation of the induced geometric tensor (metric, Christoffel symbols, curvature, torsion, etc).

Talk 2 We study of the harmonic flow of H-structures. This is the negative gradient flow of a Dirichlet-type energy functional on an isometric class of H-structures on a closed Riemannian n-manifold. Using a Bianchi-type identity for H-structures, we are able to prove monotonicity formulas for scale-invariant local versions of the energy, similar to the classic formulas proved by Struwe and Chen [2, 4] in the theory of harmonic map heat flow. We then deduce a general ϵ-regularity.

Talk 3 As a first application of the ϵ-regularity, we get long-time existence and finite-time singularity results in parallel to the classical results proved by Chen-Ding [1] in harmonic map theory. In particular, we show that if the energy of the initial H-structure is small enough, depending on the L^∞-norm of its torsion, then the harmonic flow exists for all time and converges to a torsion-free H-structure. Moreover, we prove that the harmonic flow of H-structures develops a finite time singularity if the initial energy is sufficiently small but there is no torsion-free H-structure in the homotopy class of the initial H-structure.

Talk 4 We improve the long-time existence and convergence of solutions for the harmonic flow of H-structures, when the initial intrinsic torsion is small enough, this allows to prove a dynamical stability of torsion-free H-structures along the harmonic flow. As a consequence, it provides a torsion gap result in the absence of torsion-free H-structures.

Reference

[1] Y.M. Chen and W.Y. Ding, *Blow-up and global existence for heat flows of harmonic maps*, Invent. Math. 99 (1990), no. 3, 567–578.

[2] Y.M. Chen and M. Struwe, *Existence and partial regularity results for the heat flow for harmonic maps*, Math. Z. 201 (1989), no. 1, 83–103.

[3] D Fadel, E. Loubeau, A. Moreno and H. Sá Earp: *Flows of geometric structures*, Journal für die reine und angewandte Mathematik (Crelles Journal), 2024(817), 67-152.

[4] M. Struwe, *On the evolution of harmonic maps in higher dimensions*, J. Differential Geom. 28 (1988), no. 3, 485–502.