The bilinear Hilbert transform is a bilinear singular integral operator (or Fourier multiplier) which is invariant not only under translations and dilations, but also under modulations. This additional symmetry turns out to make proving $L^p$-bounds especially difficult. I will give an overview of how time-frequency analysis is used in proving these $L^p$-bounds, with focus on the recently-understood setting of functions valued in UMD Banach spaces.