

JAE - School of Mathematics
Coarse geometry, Groups and Operator Algebras

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Abstract

In topology and geometry one is typically interested in the local behavior of the space, i.e., in how it looks like under the microscope. *Coarse geometry* is opposite to this point of view; it is the study of metric spaces from an asymptotic point of view, i.e., looking at the space from a great distance. In this sense a ball looks like a point and \mathbb{R}^n may be identified \mathbb{Z}^n , $n \in \mathbb{N}$. Besides geometry and topology, these ideas have also entered other areas of mathematics like group theory or the theory of operator algebras, i.e., algebras of bounded and linear operators in a complex Hilbert space. Coarse geometry allows to look at a finitely generated group Γ as a geometrical object. Moreover, the class of uniform Roe algebras provide a natural link between metric spaces and operators.

In this course I will present the ideas mentioned above and work them out through many examples. We will also solve problems in the final part of the lecture and try to encourage interaction. In the lectures I will describe fundamental notions like quasi-isometries or property A. We will look at finitely generated groups as metric spaces and study amenability of groups from a large scale perspective. Finally, we will define the uniform Roe algebra associated to a metric space. We will see how properties of the metric space are translated into operator algebraic properties.

The course will be structured as follows:

Index:

Chapter 0: Motivation: basic intuition and the Milnor-Švarc lemma.

Chapter 1: Metric spaces and quasi-isometries

Chapter 2: Amenability of groups and property A of metric spaces.

Chapter 3: The uniform Roe algebra of a metric space.

References:

1. N.P. Brown and N. Ozawa, *C*-algebras and Finite Dimensional Approximations*, American Mathematical Society, Providence, Rhode Island, 2008.
2. T.G. Ceccherini-Silberstein and M. Coornaert, *Cellular Automata and Groups*, Springer-Verlag, Berlin, 2010.
3. C. Löh, *Geometric Group Theory. An Introduction*, Springer Verlag, 2017.
4. P.W. Nowak and G. Yu, *Large Scale Geometry*, European Mathematical Society, Zürich, 2012.
5. J. Roe, *Lectures on Coarse Geometry*, American Mathematical Society, Providence, Rhode Island, 2003.
6. J. Roe, *What is a coarse space?*, Notices of the AMS **53** (2006) 668-669.
7. R. Willett, *Some notes on property A*, In *Limits of graphs in group theory and computer science*, G. Arzhantseva and A. Valette (eds.), EPFLPress, Lausanne, 2009 p. 191-281.