2020 JAE School of Mathematics - ICMAT

Roots of random polynomials

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Abstract

Motivated by the need to understand particles with a local repulsion, the following model has been proposed: Consider a polynomial with independent random coefficients and we compute its zeros in the complex plane. This is easily simulated numerically. The careful choice of random coefficients allows us to obtain a random distribution of zeros which are invariant under translations in the plane or Möbius transforms in the unit disk. We will study the interplay between complex analysis, probability, and potential theory. The lectures will cover:

1. Gaussian Analytic Functions.
2. Isometry Invariant Zero sets:
   (a) Fock space and the complex plane with the Euclidean metric.
   (b) Bergman space and the unit disk with the hyperbolic metric.
   (c) Weyl polynomials and the Riemann sphere with the Fubini metric.
   (d) The Paley-Wiener space and the real line with the Euclidean metric.
4. Calabi’s rigidity.

References:

2. Fedor Nazarov, Mikhail Sodin, What is... a Gaussian Entire Function?, Notices of the AMS (March, 2010), Volume 57, Number 3, pp. 375 - 377.