JAE School of Mathematics 2019

Instituto de Ciencias Matemáticas (ICMAT)
Madrid, Spain
10–21 June 2019

ICMAT
INSTITUTO DE CIENCIAS MATEMÁTICAS

EXCELENCIA
SEVERO
OCHOA

Contents

The main goal of the JAE School of Mathematics 2019 is to provide a space for interaction between high-level researchers and students who enjoy doing mathematics in an ideal research environment.

The school will take place at **Instituto de Ciencias Matemáticas** (Madrid, Spain). The activity will start on Monday 10th of June and will end on Friday 21st of June 2019.

The organisers wish to thank you for your interest and participation in this school.

Web page: `https://www.icmat.es/events/JAESchool/programme2019`

1 Speakers

María Barbero (UPM-ICMAT)

Pablo Candela (ICMAT-UAM)

Joana Cirici (UB)

Javier Gomez Serrano (Princeton)

Conchita Martinez (Zaragoza)

David Martin (ICMAT-CSIC)

José M. Manzano (UCM)

2 Venue and meals

The meeting will take place at the **Instituto de Ciencias Matemáticas**, Madrid, Spain.

Instituto de Ciencias Matemáticas
Address: Nicolas Cabrera 13–15,
Campus de Cantoblanco, UAM, 28049 Madrid, SPAIN.
Web: http://www.icmat.es/
Phone: +34 91 2999 704

From Monday to Friday, lunch will be provided at Plaza Mayor, on the Campus de Cantoblanco UAM. A social tapas dinner will take place on a secret time and location, with costs covered independently by each participant. No other meals will be arranged by the organisation.

Information on how to get to the venue can be found at the ICMAT web site:
http://www.icmat.es/facilities/howtoarrive

There is Wi-Fi connectivity available throughout ICMAT.

3 Organising Committee

Yago Antolín Pichel (ICMAT-UAM)

Mario Garcia-Fernandez (ICMAT-UAM)

4 Sponsors

ICMAT Severo Ochoa Programme

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5 Programme

María Barbero & David Martin: *Geometric methods for robotics*

Pablo Candela: *Additive combinatorics, Gowers norms and Nilspaces*

Joana Cirici: *Topology of complex varieties*

Javier Gomez Serrano: *Computer-assisted proofs in analysis*

Conchita Martinez: *Group cohomology*

José M. Manzano: *Surfaces of constant mean curvature*

6 Schedule

1ª semana

<table>
<tr><th></th><th>lunes</th><th>martes</th><th>miércoles</th><th>jueves</th><th>viernes</th></tr>
<tr><td>9:30-10:00</td><td rowspan="4">COM</td><td rowspan="4">COM</td><td rowspan="4">COM</td><td rowspan="4">COM</td><td rowspan="4">COM</td></tr>
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Curso	Código	Teoría	Problemas
Combinatoria aditiva, normas de Gowers, y nilespacios	COM		
Cohomología de grupos	CHG		
Superficies de curvatura media constante	CMC		

Pausa

Comida

2ª semana

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<tr><td>16:30-17:00</td></tr>
<tr><td>17:00-17:30</td><td>Pausa</td><td rowspan="3"></td></tr>
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<tr><td>18:00-18:30</td></tr>
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Curso	Código	Teoría	Problemas
Topología de las variedades complejas	TVC		
Métodos geométricos para robótica	MGR		
Computer-assisted proofs in analysis	CAP		

Pausa

Comida

Image by A. Pereira

7 Abstracts

María Barbero & David Martin: *Geometric methods for robotics*

For many years, robotics and mathematics evolved as two different disciplines. It was not until the seventies that it became clear the benefits of using mathematics, especially differential geometry, to handle problems in robotics. The mathematical community made a great effort to disseminate differential geometry to engineers. The key element to describe the motion of underwater vehicles, drones, manipulators is Lie groups. The configuration spaces associated with those mechanical systems are Lie groups. For instance, Lie groups describe the rigid-body motions in the plane and in the space. In recent years, that geometric structure appears in paper written by engineers. What acknowledges the advantages of use them. In this course we will introduce Lie groups and Lie algebras to show how they naturally appear when describing configuration spaces of many mechanical systems. Then controls will be introduced so that we can act on the mechanical systems with a specific purpose, such as optimization of a particular cost function (time, energy, etc). The differential equations describing the motion of the systems are often impossible to be exactly solved. Numerical integration is needed to solve the systems and run simulations. Here, geometry also contributes into creating geometric integrators that preserve interesting properties of the systems such as the symplectic structure, etc. The contents of the course are the following ones:

1. Lie groups and Lie algebras.
2. Configuration spaces for manipulators and rigid body.
3. Optimal control problems on Lie groups.
4. Introduction to numerical integration.
5. Multi-agent systems and hybrid systems.

This course intends to provide you with a few glimpses of how geometry plays a crucial role in robotics.

References:

1. R. Abraham, J. E. Marsden, Foundations of Mechanics, Addison-Wesley, 1987.
2. A. A. Agrachev, Y. Sachkov, Control Theory from the Geometric Viewpoint, Springer Verlag Berlin Heidelberg, 2004.
3. S. Blanes, F. Casas, A concise introduction to geometric numerical integration, Monographs and Research Notes in Mathematics, CRC Press, Boca Raton, FL, 2016.
4. J. J. Craig, Introduction to Robotics: Mechanics and Control, 3rd ed, Pearson Prentic Hall, Upper Saddle River, NJ, 2004.
5. E. Hairer, C. Lubich, G. Wanner, Geometric Numerical Integration, volume 31 of Springer Series in Computational Mathematics, Springer, Heidelberg, 2010.
6. V. Jurdjevic, Geometric Control Theory, Cambridge University Press, 1996.

7. K. M. Lynch, F. C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017.
8. R. M. Murray, Z. Li, S. S. Sastry, A Mathematical Introduction to Robotic Manipulation, CRC Press, Boca Raton, CA, 1994.
9. J. M. Selig, Geometrical Methods in Robotics, Monographs in Computer Science, Springer-Verlag New York, 1996.
10. F. W. Warner, Foundations of Differentiable Manifolds and Lie groups, Graduate Texts in Mathematics, Springer-Verlag New York, Vol. 94, 1983.

Pablo Candela: *Additive combinatorics, Gowers norms and Nilspaces*

This course provides an introduction to arithmetic combinatorics, a very active area in which ideas and techniques from various fields, such as harmonic analysis, graph theory, ergodic theory or probability theory, are combined to study the structure of subsets of abelian groups. We shall approach this area through a particularly intriguing development that it has generated in the last two decades, namely the analysis of Gowers norms and its applications related to Szemerédi's theorem.

Joana Cirici: *Topology of complex varieties*

Introduciremos la noción de estructura compleja en una variedad diferencial y revisaremos propiedades básicas, ejemplos, construcciones y problemas abiertos sobre la topología de las variedades complejas. Haremos especial hincapié en aspectos de la cohomologa de dichos espacios. También, haremos una incursión en el mundo de las variedades algebraicas complejas.

Javier Gomez Serrano: *Computer-assisted proofs in analysis*

Even though it is not possible to calculate exactly a sum between two arbitrary floating point numbers, we are still able to prove rigorous theorems using the help of a computer. For the first half of the course, we will be going through the book "Validated Numerics", by Warwick Tucker (Princeton University Press). In the second half of the course, we will discuss a selection of certain theorems in ODE, PDE and other problems arising in mathematical physics from recent papers. This also includes open problems.
Programming in a high-level language is required (at the very least, the ability to read code is fundamental) since the lectures will include a practical component. Some analysis background is helpful, although the choice of which lectures to give can be slightly tailored to the students' background and interests.

Conchita Martinez: *Group cohomology*

We will review some basic notions of homological algebra, using the case of groups as the main motivating example. A (temptative) program will be:

1. Presentations of groups. Cayley graphs and Cayley complexes. Actions of groups on CW-complexes.

2. Projective and free modules and resolutions. Ext functors and basic properties. Construction of resolutions using group actions.

3. Finiteness conditions: types FP, FP_∞, cohomological dimension. Examples.

4. Spectral sequences: Lyndon-Hochschild-Serre spectral sequence.

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José M. Manzano: *Surfaces of constant mean curvature*

La curvatura media H de una superficie Σ en el espacio euclídeo $\mathbb{R}^3$ nos da una idea de cuánto se curva Σ en cada punto desde el punto de vista de un observador externo a la superficie. El estudio de superficies de curvatura media constante (y, en particular, el de las superficies mínimas con $H \equiv 0$) se ha desarrollado durante los últimos 250 años y es un tema de investigación actualmente muy activo, con aplicaciones a la física, la ingeniería y la arquitectura. Dicho estudio ha contribuido al desarrollo de muchas ideas matemáticas durante el siglo XX, tales como el análisis complejo, las ecuaciones en derivadas parciales, el cálculo de variaciones o la teoría geométrica de la medida. En este curso pondremos de manifiesto algunas de las características variacionales, analíticas (reales y complejas), geométricas y computacionales de las superficies de curvatura media constante. Las cinco sesiones tratarían los siguientes temas:

1. Breve repaso de geometría de superficies. Definición de curvatura media. Ejemplos de superficies mínimas y de curvatura media constante. Caracterización como puntos críticos del funcional área. Aplicaciones en ciencia e ingeniería.

2. Las superficies de curvatura media constante como soluciones de EDPs elípticas. El principio del máximo.

3. De las superficies de curvatura media constante al análisis complejo. Parametrizaciones isotermas. La representacióon de Weierstrass para superficies mínimas.

4. Tres problemas clásicos: Hopf, Alexandrov y Bernstein.

5. Manipulación de superficies de curvatura media constante mediante el software libre Surface Evolver. Resolución numérica del problema de Plateau. Resolución numérica del problema isoperimétrico y de problemas con pompas múltiples. Para la última sesión, los participantes en el curso podrían traer sus ordenadores portáatiles. En el curso se indicaría cómo instalar y configurar el software necesario.

Bibliografía

1. K. Brakke. Manual de usuario del software Surface Evolver, v. 2.70. Disponible online en `http://facstaff.susqu.edu/brakke/aux/downloads/manual270.pdf`

2. T.H. Colding, W.P. Minicozzi. A Course in Minimal Surfaces. Graduate Studies in Mathematics 121, AMS, Providence, Rhode Island, 2011.

3. K. Kenmotsu. Surfaces with constant mean curvature. Translations of Mathema- tical Monographs, AMS, Providence, Rhode Island, 2003. isbn: 978082183479-4.

4. R. Osserman. A survey of minimal surfaces. Dover Publicantions, Inc. Mineola, New York, 1986. isbn: 978048649514-0.

5. J. Pérez. Superficies mínimas y de curvatura media constante. Disponible online en `http://wpd.ugr.es/ jperez/wordpress/wp-content/uploads/todo-1.pdf`