INTRODUCCIÓN A LA INVESTIGACIÓN SEVERO OCHOA 2026
RESEARCH PROJECT: BANACH SPACES AND BANACH LATTICES

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Various spaces of functions are ubiquitous in Analysis and Applications, which lead to function spaces as the central objects of study of Functional Analysis. A Banach space is an abstraction of a function space, hence Banach space theory has been central for Functional Analysis. However, while Banach space theory captures the algebraic and the metric structures of function spaces, it "does not see" their order structures.

To fix this issue, one is lead to introduce Banach lattices, which are a class of Banach spaces incorporating order and lattice structure. Many of the classical function spaces arising in analysis are Banach lattices, this includes for instance sequence spaces ℓ_p, spaces of integrable functions on a measurable space $L_p(\Omega, \Sigma, \mu)$ and spaces of continuous functions on a compact space $C(K)$. Research in this area focuses on the interplay between algebraic, geometric, analytic and order structures.

In the recent years, the notion of free Banach lattice generated by a Banach space has been introduced. This provides a new canonical functor between the categories of Banach spaces (and linear operators) and Banach lattices (with lattice homomorphisms). This new tool has become fundamental to tackle some exciting open problems in the area. The student will have the chance to approximate to this very recent developements in Functional Analysis and learn from a variety of topics connected to them.

In order to properly profit from this project it is necessary to have gone through a basic course in Functional Analysis.

References

[1] A. Avilés, J. Rodríguez, P. Tradacete, The free Banach lattice generated by a Banach space. J. Funct. Anal. 274 (2018), 2955-2977.

[2] P. Meyer-Nieberg. Banach lattices. Universitext. Springer-Verlag, Berlin, 1991.

[3] T. Oikhberg, M.A. Taylor, P. Tradacete, V.G. Troitsky, Free Banach lattices. J. Eur. Math. Soc. (JEMS) (to appear)