Symplectic integrators: an introduction to Hamiltonian mechanics

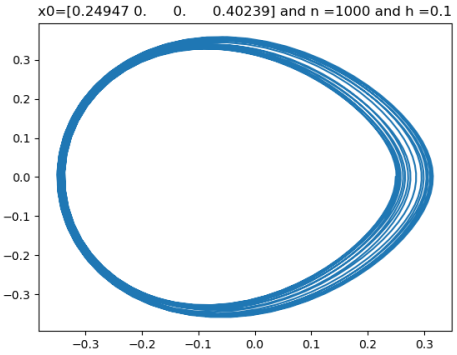
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The Hamiltonian formalism plays a major role in mechanics, describing a wide range of dynamical systems in physics: celestial mechanics, oscillators and pendula of many kinds, waves, etc. It is also a powerful tool to describe symmetries and invariants of those dynamical systems, because the underlying geometry, called symplectic geometry, is very rich.

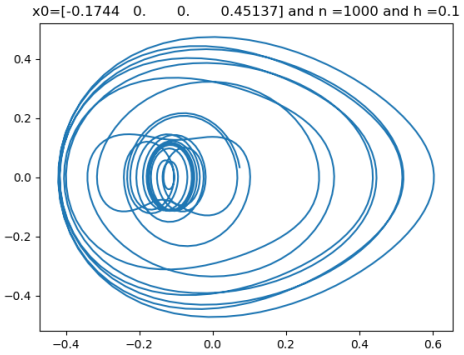
During this internship, we propose to study several examples of ordinary differential equations admitting a Hamiltonian formulation, starting from elementary examples: the harmonic oscillator or the hyperbolic pendulum for instance. For those simple examples, one can prove the existence of periodic orbits and observe the behavior of various numerical methods with respect to this periodicity. Some numerical methods are known to destroy this periodicity, and some others are known to behave well with respect to it. One goal of this internship is to understand why.

For those who wish to go further, this internship can lead to an introduction to notions such as integrable systems, KAM theory, elliptic and hyperbolic singularities, and various other concepts of the theory of dynamical systems.

A minimal knowledge of differential calculus, as well as some elementary notions of programming, are required for this internship. It can fit one student alone, but can also be led by a group of two or three students working together. The internship has a duration of 1 month and will be hosted at ICMAT in Madrid, from the 22nd of June to the 31st of July approximately.



A quasi-periodic orbit



An orbit exhibiting hyperbolic properties

Numerical simulations of the Hénon-Heiles dynamical system using a symplectic integrator

BIBLIOGRAPHY

- *Geometric Numerical Integration*, E. Hairer, C. Lubich and G. Wanner, Springer Series in Computational Mathematics, 2006.
- *Preservation and destruction of periodic orbits by symplectic integrators*, R. I. McLachlan, and D. R. J. O'Neale, Numerical Algorithms, 2010.
- *Some stability properties of Hamiltonian Poisson integrators*, O. Cosserat, arXiv:2511.14303, 2025.