ASPHERICAL 4-MANIFOLDS WITH POSITIVE EULER CHARACTERISTIC

CATERINA CAMPAGNOLO, UAM

Aspherical manifolds form a rich class of manifolds, where group theory can interplay interestingly with topological and geometrical tools. Dimension 4 is special among manifolds, as many results that are true in all higher dimensions do not hold there, and as the good understanding we have of manifolds in dimensions up to 3 stops at dimension 4. Thus many open questions are awaiting among 4-manifolds.

In this project, we propose to read the recent preprint *Aspherical 4-manifolds with positive Euler characteristic and their geography* by Pietro Capovilla (arXiv:2511.15577), where a family $\{M_n\}_{n\in\mathbb{N}}$ of aspherical manifolds is constructed, having equal Euler characteristic and signature: $\chi(M_n) = \sigma(M_n) = n$, $\forall n \in \mathbb{N}$.

This family allows to test several properties and inequalities known to be satisfied by certain families of 4-manifolds. In particular, we will study it in the light of the *simplicial volume*. This is a topological invariant introduced by Gromov in the '80s that proved to have many geometrical applications.

A famous open question concerning simplicial volume is whether aspherical manifolds with vanishing simplicial volume also have vanishing Euler characteristic. The question is solved in dimension 2 and trivial in dimension 3, making dimension 4 the first relevant dimension where to study it. We will be interested in the strategies opened by this article to approach this question.

Date: February 2026.