INTRODUCCIÓN A LA INVESTIGACIÓN SEVERO OCHOA 2024
RESEARCH PROJECT: POSITIVITY IN OPERATOR THEORY

SUPERVISOR: PEDRO TRADACETE

Ordered structures are a common framework for a myriad of mathematical problems. In an ordered space X (either finite dimensional like $\mathbb{R}^n$, or infinite dimensional like a space of real valued functions on some larger set) a linear operator $T : X \to X$ is called positive when it maps the positive cone $X_+ = \{x \in X : x \geq 0\}$ into itself. This induces an ordered structure at the level of operators by setting $S \leq T$ whenever $T - S$ is positive.

A surprising feature of this ordered structure is that it is linked to several seemingly unrelated topological notions. One can wonder for instance under which conditions a certain topological property of a positive operator T, is inherited by every operator satisfying $0 \leq S \leq T$. We will explore the many properties that satisfy this sort of scheme, including compactness, singularity and spectral properties. Several open problems in this area will allow the student to grasp up-to-date research and learn ideas from spectral theory, operator ideals and Banach lattices.

In order to properly profit from this project it is highly recomended to have gone through a basic course in Functional Analysis.

REFERENCES

[1] Y. A. Abramovich, C. D. Aliprantis, An invitation to operator theory. Graduate Studies in Mathematics, 50. American Mathematical Society, Providence, RI, 2002.

[2] J. Flores, F. L. Hernández, P. Tradacete, Domination problems for strictly singular operators and other related classes. Positivity 15 (2011), no. 4, 595–616.