

Tutor :

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<https://www.icmat.es/miembros/nfreitas/>

Method :

Online meetings.

Language :

English, Spanish or Portuguese

Project Title :

Haar measures and Arithmetic Groups

Summary :

A Haar measure allows to define integrals for functions on locally compact topological groups. This idea was introduced by Alfréd Haar in 1933 and is used in many parts of mathematics as analysis, representation theory or number theory. In number theory, for example, it plays an essential rôle in the basic theory of arithmetic groups.

Arithmetic groups arise naturally in the study of arithmetic properties of quadratic forms and other classical topics in number theory. They also play a central rôle in the subject of automorphic forms and the Langlands program which is a fundamental research subject in modern number theory. They are also used to construct examples of Riemannian manifolds and hence are objects of interest in differential geometry and topology.

The purpose of this project is to give the student an introduction Haar measures and briefly discuss arithmetic groups. We start by covering the necessary preliminaries on topological groups and measure theory, the Haar correspondence, unimodularity and Haar measures on coset spaces.

Then we will do a brief introduction to algebraic groups over \mathbb{Q} and arithmetic subgroups, describing Siegel sets and reduction theory (illustrating via the reduction theory of quadratic forms). Finally, depending on time availability we may state and overview the ideas of some fundamental results like the Borel-Harish-Chandra theorem, Godement's compactness criterion, and Borel's density theorem. We might also discuss co-volumes of arithmetic subgroups and masses of quadratic lattices, and we will study example of arithmetic hyperbolic 3-manifold.