

INTRODUCCIÓN A LA INVESTIGACIÓN SEVERO OCHOA 2022
RESEARCH PROJECT: DOUBLING CONSTANTS IN METRIC SPACES AND GRAPHS

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The aim of this project is to illustrate a fruitful meeting point between geometry and analysis. It is well-known that the Lebesgue measure on \mathbb{R}^n satisfies the identity $\lambda(B(x, r)) = r^n \lambda(B(x, 1))$, where $B(x, r)$ denotes the open ball of center x and radius r . A natural way to generalize this notion to more abstract geometrical settings is the following: let us say a measure μ on a metric space is doubling if there is a constant $C > 0$ such that

$$\mu(B(x, 2r)) \leq C\mu(B(x, r)).$$

The best possible C one can find in the above inequality is called the *doubling constant* of the measure μ , and it is interesting to investigate how small this can be among all measures defined on a given metric space.

In this research project, the student will be able to check by elementary methods that the doubling constant is always greater than or equal to 2, and will approach the computations of these constants for specific metric spaces, with particular focus on the discrete case of finite and infinite graphs.

Only basic notions of metric spaces and measure theory are required.

REFERENCES

- [1] J. Heinonen: Lectures on Analysis on Metric Spaces, Universitext, Springer-Verlag, New York, 2001.
- [2] E. Durand-Cartagena, J. Soria, P. Tradacete: Doubling constants and spectral theory on graphs, (preprint).
- [3] J. Soria, P. Tradacete: The least doubling constant of a metric measure space. Ann. Acad. Sci. Fenn. Math. 44 (2019), 1015-1030.