Introduction to research in the Hitchin-Ngo ICMAT Laboratory

We propose two research themes that are part of the main objectives of the Hitchin–Ngo Laboratory at ICMAT. The research will be supervised by Oscar García-Prada, coordinator of the laboratory, and may involve other members of the laboratory including the chairs, Nigel Hitchin (University of Oxford, Shaw Prize in Mathematics) and Ngo Bao Chau (University of Chicago, Fields Medal). The themes fall in the are of study of moduli spaces.

Moduli spaces arise naturally in classification problems in geometry. Typically, one has a collection of objects and an equivalence relation, and the problem is to describe the set of equivalence classes. Usually, there are discrete invariants that partition this set in a countable number of subsets. In most cases there exist continuous families of objects, and one would like to give the set of equivalence classes some geometric structure to reflect this fact. This is the object of the theory of moduli spaces. The word moduli is due to Riemann, who used it as a synonym for parameters when showing that the space of equivalence classes of Riemann surfaces of a given genus $g > 1$ depends on $3g - 3$ complex numbers. After this, the concept of moduli has been used in geometry in a rather loose sense to measure variations of geometric structures of one kind or another, but it has not been however until the 1960s that one has been able to formulate moduli problems in precise terms and in some cases to obtain solutions to them.

The two research themes are the following:

- **Introduction to moduli spaces: Higgs bundles**. The goal of this project is to introduce the student in the important subject of moduli spaces providing the necessary background in topology, differential and algebraic geometry and the theory of Lie groups. The focus will be on the moduli spaces of Higgs bundles. Since the introduction of Higgs bundles by Hitchin almost 35 years ago, the moduli spaces of these objects have played a central role in many areas of mathematics and theoretical physics. In particular they were central in the proof of the Fundamental Lemma of Langlands Theory that won Ngo a Fields Medal in 2010.

- **Introduction to moduli spaces: Character varieties**. The goal of this project is to introduce the student in the important subject of moduli spaces providing the necessary background in topology, differential and algebraic geometry and the theory of Lie groups. The focus will be on the moduli spaces of representations of the fundamental group of a surface in a Lie group, the so-called character varieties. These varieties play a central role in many areas of mathematics and theoretical physics, having led in particular to higher Teichmüller theory, a very active area of international research.

Para más información, consultar: https://www.icmat.es/miembros/garcia-prada

Número máximo de estudiantes a supervisar: 2