# Introducción a la investigación

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Se proponen dos proyectos en la intersección de la Geometría Diferencial y las Ecuaciones en Derivadas Parciales:

1. **The Keller-Segel system**.

   The Keller-Segel system in $\mathbb{R}^2$ is the classical diffusion model for *chemotaxis*, the motion of a population of bacteria driven by standard diffusion and a nonlocal drift given by the gradient of a chemoatractant, a chemical the bacteria produce. Depending on the value of the total mass, the equation could have a blow-up behavior (aggregation) or spread out to zero (diffusion). At the precise critical value, the resulting equation is important in geometry, since it describes a (positive) constant Gauss curvature surface.

2. **A phase field formulation of the Willmore problem**.

   From the differential geometry point of view, the Willmore energy measures how much a given surface deviates from a round sphere. It is defined to be the integral of the square of the mean curvature minus the Gaussian curvature. From the point of view of applications, cell membranes tend to position themselves so as to minimize Willmore energy, and these dynamics are usually modelled using phase field methods. We would like to: first, understand the Willmore energy in differential geometry and, second, to study the convergence of such a phase field formulation.