Simulation of dynamical systems using variational integrators

Leonardo Colombo

In many engineering applications, numerical integrators for continuous-time equations of motion of physical systems are usually derived by discretizing differential equations. However, the inherent geometric structure of the governing continuous-time equations and conserved quantities are not preserved in simulations with the traditional integrators.

Variational integrators are numerical methods derived from the discretization of variational principles. These integrators retain some of the main geometric properties of the continuous-time systems, such as symplecticity, momentum conservation, and also exhibit easily checkable behavior of the energy associated to the system. This class of numerical methods have been applied to a wide range of problems in optimal control, constrained systems, power systems, nonholonomic systems, and systems on Lie groups.

The goal of the research stage is to learn and implement variational integrators for some classes of mechanical systems useful in robotics applications. The objectives to reach along the research stage are:

- Learn the basic results about Geometric Integration and variational integrators.
- Simulation of dynamical (mechanical) systems using variational integrators.
- Learn about optimal control of mechanical systems.
- Simulation of solutions for optimal control problems using appropriated variational integrators.

It is recommendable some knowledge about ordinary differential equations and numerical methods for ODE's. The student should be familiarized (mandatory) with the use of Python or Matlab for ordinary differential equations with practice at least in the homework assignments of courses of his/her corresponding degree.

References

[1] E. Hairer, C. Lubich and G. Wanner. *Geometric Numerical Integration, Structure-Preserving Algorithms for Ordinary Differential Equations.*

Springer Series in Computational Mathematics, Springer-Verlag, Berlin, **31**, (2002).

[2] M. Kobilarov, and J. Marsden (2011). Discrete geometric optimal control on lie groups. IEEE Transactions on Robotics, 27(4), 641–655.

[3] J. Marsden and M. West. *Discrete Mechanics and variational integrators.* Acta Numerica, **10**, pp. 357–514, (2001).

[4] S. Ober-Blöbaum, O. Junge, and J. Marsden (2011). Discrete mechanics and optimal control: an analysis. ESAIM: Control, Optimisation and Calculus of Variations, 17(2), 322–352.