Numerical computations for the Calderón problem
Part 1

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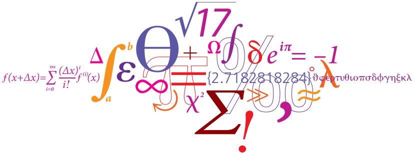
Outline

1. The Calderón problem
2. Calderón's linear reconstruction algorithm
3. The direct, non-linear reconstruction algorithm in 2D
4. Solution in 2D - regularized Dbar method
5. Conclusions



Collaborators

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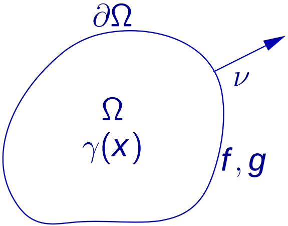
1. The Calderón problem

The conductivity equation

Smooth bounded domain $\Omega \subset \mathbb{R}^n,\ n = 2, 3$; conductivity coefficient $\gamma \in L^\infty(\Omega),\ C^{-1} \le Re(\gamma)$, for $C > 0$.

A voltage potential u in Ω generated by Dirichlet data f

$$\nabla \cdot \gamma \nabla u = 0 \text{ in } \Omega.$$
$$f = u|_{\partial\Omega}$$



Corresponding Neumann data:

$$g = \gamma \partial_\nu u|_{\partial\Omega}.$$

Dirichlet to Neumann map

$$\Lambda_\gamma : H^{1/2}(\partial\Omega) \to H^{-1/2}(\partial\Omega)$$
$$f \mapsto g.$$

Inverse problem

Consider the (non-linear) mapping

$$\Lambda \colon \gamma \mapsto \Lambda_\gamma.$$

Conductivity $\rightarrow$ Boundary measurements

This mapping encodes the direct problem.

The Calderón problem (the inverse conductivity problem):

- Uniqueness: is Λ injective?
- Reconstruction: how can γ be computed from Λ_γ?

Data for inverse problem contains infinitely many measurements with infinite precision.

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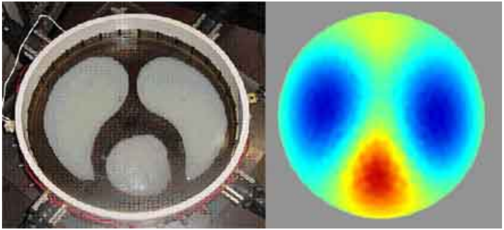
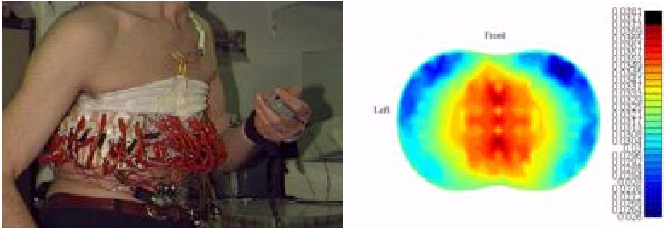
Applications include Electrical Impedance Tomography, emerging technology for medical imaging.

Electrical Impedance Tomography

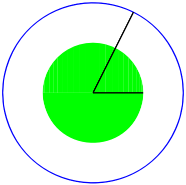
Tank experiment:



Clinical experiment



Most simple example



Suppose $\Omega = \mathbb{D} \subset \mathbb{R}^2$ and let

$$\gamma(x) = \begin{cases} \gamma_0 & |x| \leq r_0 \\ 1 & r_0 < |x| \leq 1. \end{cases}$$

Then one can show that

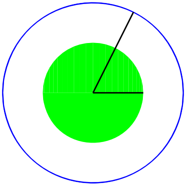
$$\Lambda_\gamma(e^{in\theta}) = \lambda_n e^{in\theta},$$

with

$$\lambda_n = |n| - \delta\lambda_n, \qquad \delta\lambda_n = 2|n| \frac{1}{1 + \frac{1+\gamma_0}{1-\gamma_0} r_0^{-2|n|}}.$$

Inverse problem: Given Λ_γ or equivalently $\{\lambda_n\}$, find γ.

Most simple example



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Inverse problem: Given Λ_γ or equivalently $\{\lambda_n\}$, find γ.
Non-linear, severely ill-posed, inverse problem

Short and incomplete history

1980 Calderón: Problem posed, uniqueness for linearized problem, and linear, approximate reconstruction algorithm

3D

1987 Sylvester and Uhlmann: Uniqueness for smooth conductivities. Implicit reconstruction algorithm

1987-88 Novikov, Nachman-Sylvester-Uhlmann, Nachman: Uniqueness for conductivities with 2 derivatives and explicit high frequency reconstruction algorithm. Multidimensional D-bar equation.

1990 Alessandrini: Stability

2003 Brown-Torres, Päivärinta-Panchenko-Uhlmann: Uniqueness for conductivities with $3/2$ derivatives.

2006 Cornean-Knudsen-Siltanen: Low frequency reconstruction algorithm

2010 Bikowski-Knudsen-Mueller: Numerical implementation of simplified reconstruction algorithm

2011 Debary - Hansen- Knudsen: Implementation of more accurate numerical reconstruction method

2D

1996 Nachman: Uniqueness and reconstruction for $W^{2,p}(\Omega)$ conductivities.

1997 Liu: Stabilty for $W^{2,p}(\Omega)$ conductivities

1997 Brown-Torres: Uniqueness for $W^{1,p}(\Omega)$ conductivities

2001 Barceló-Barceló-Ruiz: Stability for $C^{1+\epsilon}$ conductivities

2001 Knudsen-Tamasan: Reconstruction for $C^{1+\epsilon}$ conductivities

2005 Astala-Päivärinta: Uniqueness and reconstruction for $L^{\infty}(\Omega)$

2009 Knudsen-Lassas-Mueller-Siltanen: Regularized $\overline{\partial}$-method

2010 Clop-Faraco-Ruiz: Stability for discontinuous conductivities

+ many more

Reconstruction methods

The reconstruction algorithms for the general Calderón problem can roughly be characterized:

1. Linearization methods
2. Optimization methods
3. Statistical methods
4. Direct methods

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1. Linearization methods
2. Optimization methods
3. Statistical methods
4. Direct methods

Other qualitative methods include:

- the probe method
- the linear sampling method
- the factorization method

Assumptions

Assume throughout that

1. Ω and γ are sufficiently smooth
2. $\gamma = 1$ near $\partial\Omega$
3. γ is extended to $\mathbb{R}^n \setminus \Omega$ by $\gamma = 1$
4. In 2D assume γ is real

Note that 1. and 4. are restrictive, but 2.-3. can be assumed WLOG.

2. Calderón’s linear reconstruction algorithm

Calderón's reconstruction method

Linearize the mapping $\Lambda\colon \gamma \mapsto \Lambda_\gamma$, i.e. write

$$\Lambda_\gamma - \Lambda_1 \approx d\Lambda(\gamma - 1).$$

Integration by parts

$$\begin{aligned}\langle(\Lambda_\gamma - \Lambda_1)f, h\rangle &= \int_{\partial\Omega}(\Lambda_\gamma - \Lambda_1)f\ h dS\\ &= \int_\Omega(\gamma - 1)\nabla u \cdot \nabla v\ dx,\end{aligned}$$

where

$$\nabla \cdot \gamma\nabla u = 0,\ u|_{\partial\Omega} = f, \quad \Delta v = 0,\ v|_{\partial\Omega} = h.$$

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where

$$\nabla \cdot \gamma \nabla u = 0,\ u|_{\partial\Omega} = f, \quad \Delta v = 0,\ v|_{\partial\Omega} = h.$$

Then it turns out

$$\langle d\Lambda(\gamma - 1)f, h\rangle = \int_\Omega (\gamma - 1)\nabla u \cdot \nabla v\ dx,$$

where

$$\Delta u = 0,\ u|_{\partial\Omega} = f, \quad \Delta v = 0,\ v|_{\partial\Omega} = h.$$

Harmonic complex exponentials

To obtain information from the linearized equation take h, f as restrictions of harmonic functions

$$f = e^{ix\cdot\zeta}, \qquad h = e^{-ix\cdot(\xi+\zeta)}$$

with $\xi \in \mathbb{R}^n$ and $\zeta = \zeta(\xi) \in \mathbb{C}^n$ s.t.

$$(\xi+\zeta)^2 = \zeta^2 = 0.$$

From Λ_γ we can compute the *near-field scattering transform* (non-linear in $\gamma - 1$)

$$\begin{aligned}\mathbf{t}^{\mathrm{exp}}(\xi,\zeta) &= \left\langle (\Lambda_\gamma - \Lambda_1)e^{ix\cdot\zeta}, e^{-ix\cdot(\xi+\zeta)} \right\rangle \\ &= \int_\Omega (\gamma(x) - 1)\nabla u^{\mathrm{exp}}(x,\zeta)\cdot\nabla e^{-ix\cdot(\xi+\zeta)}dx,\end{aligned}$$

with

$$\nabla\cdot\gamma\nabla u^{\mathrm{exp}} = 0 \text{ in } \Omega, u^{\mathrm{exp}}|_{\partial\Omega} = e^{ix\cdot\zeta}.$$

Linear approximation

Writing $u^{\rm exp} = e^{ix\cdot\zeta} + \delta u$ yields

$$\mathbf{t}^{\rm exp}(\xi,\zeta) = -\frac{|\xi|^2}{2}\int_\Omega (\gamma(x)-1)e^{-ix\cdot\xi}dx + R(\xi,\zeta),$$

$$R(\xi,\zeta) = \int_\Omega (\gamma(x)-1)\nabla\delta u\cdot\nabla e^{-ix\cdot(\zeta+\xi)}dx.$$

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The linear part:

$$\langle d\Lambda(\gamma-1)f,h\rangle = -\frac{|\xi|^2}{2}\int_\Omega (\gamma(x)-1)e^{-ix\cdot\xi}dx.$$

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Estimates:

$$|R(\xi,\zeta)| \le C\|\gamma-1\|^2_{L^\infty(\Omega)}(1+|\zeta|)^2 e^{2R|\zeta|}, \qquad \Omega\subset B_R$$
$$|R(\xi,\zeta)| = \mathcal{O}(|\xi|^2) \text{ for } \xi \text{ small.}$$

The reconstruction formula

Calderón approximation formula (with low pass filter):

$$\gamma^{\mathrm{app}}(x) = 1 - \frac{1}{2(2\pi)^n} \int \frac{\mathbf{t}^{\mathrm{exp}}(\xi,\zeta)}{|\xi|^2} e^{ix\cdot\xi} \chi_K(\xi) d\xi.$$

Solves the problem to first order. Estimate:

$$\begin{aligned} \|\gamma^{\mathrm{app}}(x) - \eta_\gamma * \gamma\|_{L^\infty} &\le \|R(k)\hat{\eta}(k/\gamma)\|_{L^1(\mathbb{R}^2)} \\ &\le C\|\gamma - 1\|^{1+\alpha}_{L^\infty(\Omega)} (\log(\|\gamma - 1\|_{L^\infty(\Omega)}))^2. \end{aligned}$$

Support theorem

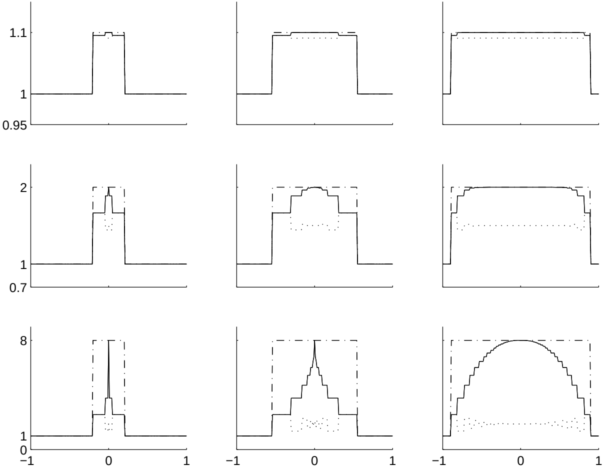
For the concentric inclusion in unit disk we obtain without filtering the linearized reconstruction

$$\gamma^{\text{app}}(x) = 1 + 2\alpha \sum_{m=0}^{\infty} \alpha^m \chi_{r^{m+1}}(x)$$

with $\alpha = \frac{\sigma-1}{\sigma+1}$.

Special case of a recent support theorem obtained by von Harrach-Seo.

Calderón's method on concentric case



3. The direct, non-linear reconstruction algorithm in 2D

Transformation to Schrödinger equation

Suppose u solves

$$\nabla \cdot \gamma \nabla u = 0 \text{ in } \Omega, \qquad u|_{\partial\Omega} = f.$$

Then $v = \gamma^{1/2} u$ solves

$$(-\Delta + q)v = 0 \text{ in } \Omega \qquad v|_{\partial\Omega} = \gamma^{-1/2} f,$$

with $q = \Delta\gamma^{1/2}/\gamma^{1/2} \Leftrightarrow (-\Delta + q)\gamma^{1/2} = 0$.

Dirichlet to Neumann map

$$\Lambda_q f = \partial_\nu v.$$

If $\gamma = 1$ near $\partial\Omega$ then $\Lambda_q = \Lambda_\gamma$.

Complex geometrical optics (CGO)

Let $\zeta \in \mathbb{C}^n$ such that $\zeta \cdot \zeta = 0$. Look for a CGO solution ψ to the problem

$$(-\Delta + q)\psi(x,\zeta) = 0 \text{ in } \mathbb{R}^n,$$
$$\psi(x,\zeta) \sim e^{ix\cdot\zeta} \text{ for large } |x| \text{ or } |\zeta|.$$

Lippmann-Schwinger-Faddeev (LSF) equation

$$\psi(x,\zeta) = e^{ix\cdot\zeta} + \int_\Omega G_\zeta(x-y)q(y)\psi(y,\zeta)dx, \qquad -\Delta G_\zeta = \delta_0,$$
$$G_\zeta \sim e^{ix\cdot\zeta}.$$

CGO solutions in 2D

In 2D $\zeta \in \mathbb{C}^2$ with $\zeta \cdot \zeta = 0$ is parametrized by $\zeta = (k, \pm ik),\ k \in \mathbb{C}$.
We will use complex notation $x = x_1 + ix_2$.
Equation for ψ or alternatively $\mu(x,k) = e^{ixk}\psi(x,k)$ is

$$4\overline{\partial}_x(\partial_x - ik)\mu + q\mu = 0.$$

Due to sharp estimates for ∂_x^{-1} (Vekua, 1962) one can obtain existence and uniqueness of CGO solutions for all $k \in \mathbb{C}$.

$\psi|_{\partial\Omega}$ at the boundary

Main Idea: use $\psi|_{\partial\Omega}$ as the boundary (voltage) fields
Suppose ζ is such that there is a unique CGO solution $\psi(x,\zeta)$. Then $\psi|_{\partial\Omega}$ satisfies for fixed $\zeta \in \mathbb{C}^n$ the boundary integral equation

$$\psi(x,\zeta) + \int_{\partial\Omega} G_\zeta(x-y)(\Lambda_\gamma - \Lambda_1)\psi(y,\zeta)d\sigma(y) = e^{ix\cdot\zeta}, \quad x \in \partial\Omega.$$

Written in terms of layer potentials

$$\psi + S_\zeta(\Lambda_\gamma - \Lambda_1)\psi = e^{ix\cdot\zeta}, \quad x \in \partial\Omega.$$

This is a solvable Fredholm equation of the second kind.

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Solving the equation is severely ill-posed!

Non-linear direct reconstruction algorithm

The key intermediate object, the *non-physical scattering transform,*

$$\mathbf{t}(\xi,\zeta) = \int_{\partial\Omega} e^{-ix\cdot(\xi+\zeta)}(\Lambda_\gamma - \Lambda_1)\psi(x,\zeta)|_{\partial\Omega} d\sigma, \qquad (\xi+\zeta)^2 = 0$$
$$= \int_{\Omega} e^{-ix\cdot(\xi+\zeta)} q(x)\psi(x,\zeta)dx.$$

t satisfies the estimate

$$|\hat{q}(\xi) - \mathbf{t}(\xi,\zeta)| = \mathcal{O}(1/|\zeta|).$$

Reconstruction algorithm

$$\Lambda_\gamma \to \mathbf{t}(\xi,\zeta) \to \gamma(x)$$

Second step depends on 2D or 3D.

Connection to Calderón reconstruction

Near-field scattering transform:

$$\begin{aligned}\mathbf{t}^{\exp}(\xi,\zeta) &= \left\langle (\Lambda_\gamma - \Lambda_1) e^{ix\cdot\zeta}, e^{-ix\cdot(\zeta+\xi)} \right\rangle \\ &= \int_\Omega e^{-ix\cdot(\xi+\zeta)} q(x) v^{\exp}(x,\zeta)\, dx,\end{aligned}$$

with $(-\Delta + q)v^{\exp} = 0$ in Ω and $v^{\exp}|_{\partial\Omega} = e^{ix\cdot\zeta}$.

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with $(-\Delta + q)v^{\exp} = 0$ in Ω and $v^{\exp}|_{\partial\Omega} = e^{ix\cdot\zeta}$.
Scattering transform:

$$\mathbf{t}(\xi,\zeta) = \left\langle (\Lambda_\gamma - \Lambda_1)\psi, e^{-ix\cdot(\zeta+\xi)} \right\rangle$$
$$= \int_\Omega e^{-ix\cdot(\xi+\zeta)} q(x) \psi(x,\zeta)\, dx,$$

where $(-\Delta + q)\psi = 0$ in $\mathbb{R}^n$ and $\psi \sim e^{ix\cdot\zeta}$ for x near ∞.

4. Solution in 2D - the regularized Dbar-method

The Dbar reconstruction algorithm

1. Solve the boundary integral equation

$$\psi + S_k(\Lambda_\gamma - \Lambda_1)\psi = e^{ixk}, \quad x \in \partial\Omega,$$

and compute

$$\mathbf{t}(k) = \int_{\partial\Omega} e^{i\bar{k}\bar{x}}(\Lambda_\gamma - \Lambda_1)\psi(\,\cdot\,, k)d\sigma.$$

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   and compute

$$\mathbf{t}(k) = \int_{\partial\Omega} e^{i\bar{k}\bar{x}}(\Lambda_\gamma - \Lambda_1)\psi(\,\cdot\,, k)d\sigma.$$

2. Evaluate $\gamma(x) = \mu(x, 0)^2$, where $\mu(x, k) = e^{-ixk}\psi(x, k)$ is found by solving the Dbar equation

$$\overline{\partial}_k \mu(x, k) = \frac{1}{4\pi\overline{k}}\mathbf{t}(k)e_{-x}(k)\overline{\mu(x, k)}, \quad k \in \mathbb{C},$$

   with $e_{-x}(k) := e^{-i(kx+\overline{k}\bar{x})}$.

Provides a non-linear direct reconstruction algorithm for $\gamma \in C^{1+\epsilon}(\overline{\gamma})$.

Regularization of the algorithm

In practice we measure Λ_γ with noise: $\tilde{\Lambda}_\gamma = \Lambda_\gamma + \mathcal{E}$

Problem: we don't know if $\tilde{\Lambda}_\gamma \in$ range (Λ), i.e. we don't know if there is $\tilde{\gamma}$ such that $\Lambda_{\tilde{\gamma}} = \tilde{\Lambda}_\gamma$.

This assumption is often made in stability estimates for the inverse problem.

Related to the notoriously difficult problem of the charaterization of range (Λ).

Non-linear regularization

Definition

A family of continuous mappings $\Gamma_\alpha : Y \to L^\infty(\Omega)$ *parameterized by* $0 < \alpha < \infty$ *is a* regularization strategy *for F if*

$$\lim_{\alpha \to 0} \|\Gamma_\alpha \Lambda_\gamma - \gamma\|_{L^\infty(\Omega)} = 0$$

for each fixed γ*. Further, a regularization strategy with a choice* $\alpha = \alpha(\varepsilon)$ *of regularization parameter as function of noise level is called* admissible *if*

$$\alpha(\varepsilon) \to 0 \text{ as } \varepsilon \to 0,$$

and for any fixed γ *the following holds:*

$$\sup_{\Lambda_\gamma^\varepsilon} \left\{ \|\Gamma_{\alpha(\varepsilon)} \Lambda_\gamma^\varepsilon - \gamma\|_{L^\infty(\Omega)} \; : \; \|\Lambda_\gamma^\varepsilon - \Lambda_\gamma\|_Y \le \varepsilon \right\} \to 0 \text{ as } \varepsilon \to 0.$$

Concrete strategy

Let $\alpha(\epsilon) = 1/R(\epsilon)$. Define $\Gamma_{\alpha(\varepsilon)}\Lambda_\gamma$ by the steps

1. Solve

$$\tilde{\varphi}(z,k) = e^{izk} - S_k(\tilde{\Lambda}_\gamma - \Lambda_1)\tilde{\varphi}, \quad |k| < R(\epsilon),$$

   and

$$\tilde{\mathbf{t}}(k) = \int_{\partial\Omega} e^{i\overline{zk}}(\tilde{\Lambda}_\gamma - \Lambda_1)\tilde{\varphi}(\cdot,k)d\sigma(z). \quad |k| < R(\epsilon)$$

2. Solve

$$\overline{\partial}_k \tilde{\mu}(x,k) = \frac{1}{4\pi\overline{k}}\tilde{\mathbf{t}}(k)e_{-x}(k)\overline{\tilde{\mu}(x,k)}, \quad k \in \mathbb{C},$$

   and compute $\Gamma_{\alpha(\varepsilon)}\Lambda_\gamma = \tilde{\gamma}(x) = (\tilde{\mu}(x,0))^2$.

Regularization theorem

Theorem
Suppose $\Lambda_\gamma^\varepsilon = \Lambda_\gamma + \mathcal{E}$ with $\|\mathcal{E}\| \leq \varepsilon < \varepsilon$. For $R(\epsilon) = -\frac{1}{10}\log(\varepsilon)$, $\Gamma_{\alpha(\varepsilon)}\Lambda_\gamma$ is an admissible regularization strategy and

$$\|\Gamma_{\alpha(\varepsilon)}\Lambda_\gamma^\varepsilon - \gamma\|_{L^\infty(\Omega)} \leq C(-\log\varepsilon)^{-1/14}.$$

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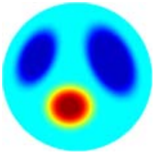
$$\|\Gamma_{\alpha(\varepsilon)}\Lambda_\gamma^\varepsilon - \gamma\|_{L^\infty(\Omega)} \leq C(-\log\varepsilon)^{-1/14}.$$

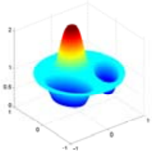
Note, we do not assume $\Lambda_\gamma^\varepsilon$ is in the range of Λ.

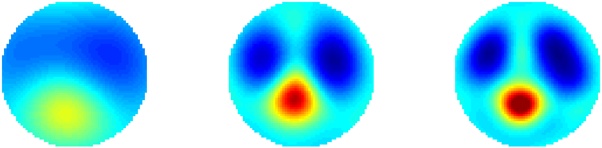
Implementation details

- Data simulated by solving forward problem by FEM
- Boundary integral equation is solved in Fourier basis and $\mathbf{t}(k)$ is computed using numerical integration
- Dbar equation solved as Lippmann-Schwinger integral equation using method of G. Vainikko

Numerical results

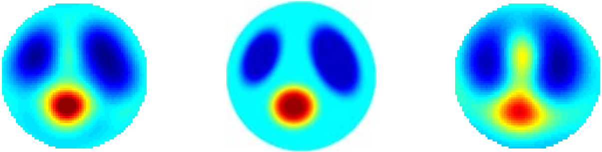






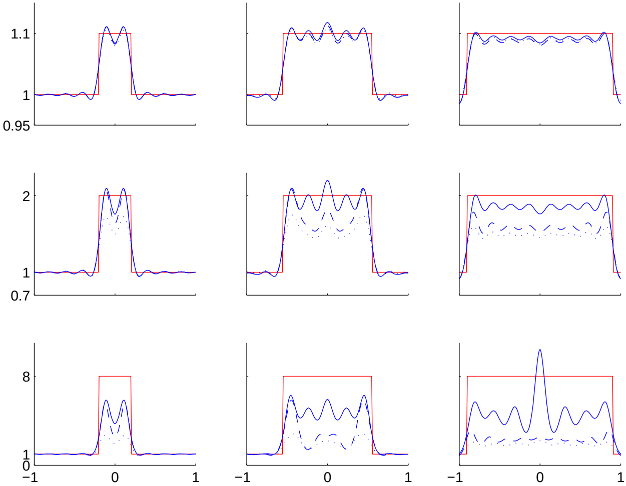
Reconstructions with noiselevel $10^{-2}, 10^{-4}$ and 10^{-6}. Error in approximation is $52\%, 14\%$ and 12% respectively.

Comparison to Calderón method



Reconstructions with noiselevel 10^{-6}. Left is non-linear direct reconstruction, right is reconstruction from Calderon method. Error in approximation is 12% and 23% respectively.

Reconstruction of non-smooth conductivities



Conclusion and outlook

- Presentation of Calderón’s linearization algorithm
- Presentation of direct non-linear reconstruction algorithm in 2D
- Connection between the 2 established
- Implementation of linear reconstruction algorithm and non-linear method for computing conductivity.
- Rigorous regularization method in 2D
- Methods work reasonable well
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- The reconstruction problem with data measured only on part of the boundary

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- The 3D problem: numerical solution of the forward problem and inverse problem

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Thank you